THE THANKSGIVING MEETING IN PASADENA

The three hundred twenty-seventh meeting of the American Mathematical Society was held at the California Institute of Technology on Saturday, November 30, 1935. The attendance included the following twenty-one members of the Society:

O. W. Albert, Harry Bateman, Clifford Bell, E. T. Bell, W. N. Birchby, Myrtie Collier, P. H. Daus, H. E. Glazier, E. R. Hedrick, G. H. Hunt, C. G. Jaeger, R. D. James, W. E. Mason, A. D. Michal, H. P. Robertson, G. E. F. Sherwood, F. C. Touton, S. E. Urner, Morgan Ward, W. M. Whyburn, E. R. Worthington.

The chair was occupied at various times by Professors Hedrick and Touton. Titles and cross references to the abstracts of the papers read at this meeting follow below; papers whose abstract numbers are followed by the letter *t* were read by title. Mr. Hall was introduced by Professor Bateman, Mr. Webb by Professor E. T. Bell, and Messrs. Elconin, Highberg, Hyers, Mersman, Paxson, and Taylor by Professor Michal.

1. *A cubic analogue of the Cauchy-Fermat theorem*, by Mr. Alvin Sugar. (Abstract No. 41-11-410.)

2. *Kinematics and world-structure*, by Professor H. P. Robertson. (Abstract No. 41-11-411.)

3. *A calculus of sequences*, by Professor Morgan Ward. (Abstract No. 41-11-412.)

4. *Complexes and manifolds represented by functions of real variables*, by Professor W. M. Whyburn. (Abstract No. 41-11-413.)

5. *On a determinant function involving the parameter of a plane curve*, by Professor Clifford Bell. (Abstract No. 41-11-414.)

6. *Distributivity of polynomial compositions*, by Professor E. T. Bell. (Abstract No. 41-11-415.)

7. *Definition of Post's generalized negative and maximum in terms of one binary operation*, by Mr. D. L. Webb. (Abstract No. 41-11-416.)

8. *Two systems of polynomials*, by Professor Harry Bateman. (Abstract No. 41-11-417.)

9. *A note on abstract polynomials in complex spaces*, by Mr. I. E. Highberg. (Abstract No. 41-11-418.)

10. *The number of representations of an integer as a sum of six*,

ten, and fourteen squares, by Dr. R. D. James. (Abstract No. 41-11-419.)

11. *Abstract integration*, by Mr. W. A. Mersman. (Abstract No. 41-11-420.)

12. *Differential properties of abstract transformation groups with abstract parameters*, by Professor A. D. Michal and Mr. Victor Elconin. (Abstract No. 41-11-421.)

13. *The solution of a second order differential system in general analysis as a function of the boundary values*, by Professor A. D. Michal and Mr. D. H. Hyers. (Abstract No. 41-11-422.)

14. *Maps of abstract topological spaces in Banach spaces*, by Professor A. D. Michal and Mr. E. W. Paxson. (Abstract No. 41-11-423.)

15. *Analytic functions in general analysis* (preliminary report), by Mr. A. E. Taylor. (Abstract No. 41-11-424.)

16. *A new class of functions of two variables involving Bessel functions of half an odd integer*, by Mr. N. A. Hall. (Abstract No. 41-11-425.)

17. *On Dickson's method of ascension*, by Mr. Alvin Sugar. (Abstract No. 41-11-426-*t*.)

18. *Postulates for special types of groups*, by Professor Raymond Garver. (Abstract No. 41-11-427-*t*.)

19. *A functional equation in arithmetic*, by Professor E. T. Bell. (Abstract No. 41-11-428-*t*.)

20. *The reciprocal of a numerical function*, by Professor E. T. Bell. (Abstract No. 41-11-429-*t*.)

21. *On certain varieties whose curve sections are hyperelliptic curves*, by Professor B. C. Wong. (Abstract No. 41-11-430-*t*.)

22. *On postulate systems for normed vector spaces*, by Mr. I. E. Highberg and Mr. A. E. Taylor. (Abstract No. 41-11-431-*t*.)

23. *An independent set of postulates for abstract linear spaces*, by Mr. I. E. Highberg and Mr. A. E. Taylor. (Abstract No. 41-11-432-*t*.)

24. *Hypercommutative rings*, by Professor A. D. Michal and Mr. Victor Elconin. (Abstract No. 41-11-433-*t*.)

E. T. BELL,
Acting Secretary

THE THANKSGIVING MEETING IN LEXINGTON

The three hundred twenty-sixth regular meeting of the Society was held at the University of Kentucky on Friday and Saturday, November 29–30, 1935. About ninety persons attended, among whom were the following thirty-two members of the Society:

N. B. Allison, B. M. Armstrong, F. R. Bamforth, I. A. Barnett, W. D. Baten, Henry Blumberg, P. P. Boyd, W. S. Claytor, L. W. Cohen, L. L. Dines, H. H. Downing, K. W. Folley, Harris Hancock, E. H. Hanson, M. H. Ingraham, E. D. Jenkins, C. G. Latimer, F. E. LeStourgeon, C. W. Mendel, H. A. Meyer, C. N. Moore, David Moskovitz, A. L. Nelson, E. G. Olds, R. S. Park, E. D. Pepper, R. F. Rinehart, J. B. Rosenbach, W. F. Smith, Guy Stevenson, J. J. Stoker, J. M. Thomas.

Friday afternoon Professor C. G. Latimer of the University of Kentucky gave an address entitled *The arithmetic of generalized quaternions*, and on Saturday afternoon Professor L. L. Dines of Carnegie Institute of Technology spoke on *Convex domains and linear inequalities*.

Friday afternoon the mathematicians and their guests attended a very enjoyable tea at the home of President and Mrs. McVey.

A dinner was held at the Lafayette Hotel Friday evening. Sixty mathematicians and their guests attended. The toastmaster, Dean P. P. Boyd of the University of Kentucky, called first upon President McVey who extended to the group a welcome to the University. A response was given by Professor C. N. Moore of the University of Cincinnati. Professor J. M. Thomas of Duke University spoke about his experiences as Managing Editor of the new Duke Mathematical Journal during its first year, and the policies and future of the Journal. Short talks were then given by Professors M. H. Ingraham of the University of Wisconsin, Harris Hancock of the University of Cincinnati, C. G. Latimer of the University of Kentucky, and L. L. Dines of the Carnegie Institute of Technology.

At the session on Saturday afternoon a resolution was passed thanking the President and Trustees of the University of Kentucky and the Department of Mathematics for their hospitality.

Friday afternoon Professor Blumberg presided. Professors Downing and Moore presided Saturday morning and afternoon respectively.

The titles of papers read at the meeting follow. Mr. Haim Reingold was introduced by Professor I. A. Barnett, Mr. P. M. Pepper by Professor Harris Hancock, and Professor Fritz John by Professor C. G. Latimer. Those abstracts numbered 1 to 3 were presented Friday afternoon; those numbered 4 to 9, Saturday morning; and those numbered 10 through 20 were read by title on Saturday morning.

1. *Generalized determinants of Vandermonde.* II, by Mr. Haim Reingold. (Abstract No. 41-11-400.)

2. *Frequency distributions of means of n independent variables whose frequency distributions can be dissected into component frequency laws*, by Professor W. D. Baten. (Abstract No. 41-11-396.)

3. *On necessary and sufficient conditions for convergence factors in multiple series*, by Professor C. N. Moore. (Abstract No. 41-11-394.)

4. *Cauchy convergence in topological space* (preliminary report), by Professor L. W. Cohen. (Abstract No. 41-11-399.)

5. *Minkowski's geometry of numbers*, by Professor Harris Hancock. (Abstract No. 42-1-67.)

6. *Application of the geometry of numbers to a generalization of continued fractions*, by Mr. P. M. Pepper. (Abstract No. 42-1-3.)

7. *Some properties of the discriminant matrices of a linear associative algebra*, by Professor R. F. Rinehart. (Abstract No. 41-11-389.)

8. *On the moments of inertia of convex regions.* II, by Professor Fritz John. (Abstract No. 41-11-402.)

9. *The form of surfaces of positive curvature in three-dimensional space*, by Professor J. J. Stoker, Jr. (Abstract No. 41-11-393.)

10. *Two notable classes of hypersurfaces*, by Professor Arnold Emch. (Abstract No. 41-11-388-*t*.)

11. *Note on the history of elliptic functions*, by Professor Tobias Dantzig. (Abstract No. 41-11-405-*t*.)

12. *A note on Young-Stieltjes integrals*, by Dr. F. G. Dressel. (Abstract No. 41-11-382-*t*.)

13. *Involutorial space transformations associated with a ra-*

tional ruled surface. II, by Professor L. A. Dye. (Abstract No. 41-11-387-*t*.)

14. *On certain factorial sums*, by Dr. Solomon Kullback. (Abstract No. 41-11-392-*t*.)

15. *Note on a problem of E. Čech*, by Dr. Leo Zippin. (Abstract No. 41-11-390-*t*.)

16. *On monotonic, complete covering systems*, by Dr. Leo Zippin. (Abstract No. 41-11-391-*t*.)

17. *A recursion formula for the polynomial solutions of differential equations with constant coefficients*, by Professor C. C. MacDuffee. (Abstract No. 41-11-395-*t*.)

18. *A new method for Waring theorems with polynomial summands.* II, by Professor L. E. Dickson. (Abstract No. 41-11-397-*t*.)

19. *Abstract vector spaces of uncountably many dimensions*, by Dr. C. W. Vickery. (Abstract No. 41-11-398-*t*.)

20. *On the moments of inertia of convex regions.* I, by Professor Fritz John. (Abstract No. 41-11-401-*t*.)

M. H. INGRAHAM,
Associate Secretary

THE ANNUAL MEETING IN ST. LOUIS

The forty-second Annual Meeting of the American Mathematical Society was held in St. Louis, Missouri, from Tuesday to Thursday, December 31, 1935–January 2, 1936. Washington University and St. Louis University acted as hosts. All sessions of the Society except the Gibbs Lecture were held in the Commerce and Finance Building of St. Louis University. The Gibbs Lecture was held in the Municipal Auditorium.

The scientific meetings were opened on Tuesday morning in two sections, one for Analysis and one for Number Theory and Algebra. On Tuesday afternoon there was a joint session of the American Mathematical Society, the Mathematical Association of America, and Section A of the American Association for the Advancement of Science. At this session Professor R. D. Carmichael delivered his retiring address as Vice-President of the American Association for the Advancement of Science and Chairman of Section A. His subject was *Linear differential equations of infinite order*. By invitation of the Committee on Program, Professor J. L. Synge gave a lecture entitled *Tensorial methods in dynamics*. Professors T. H. Hildebrandt and Solomon Lefschetz presided in turn at this session.

There were general sessions on Wednesday morning and afternoon. On Wednesday afternoon Professor Gabriel Szegö, by invitation of the Committee on Program, addressed the Society on *Some recent investigations concerning sections of trigonometric and related series*. Professor Lefschetz presided.

At a joint session on Statistics with the Econometric Society and the Institute of Mathematical Statistics on Thursday morning, Professor Thomas Rawles gave an address entitled *Mathematical theory of index numbers*. Professor H. L. Rietz presided. There was also a section on Analysis at this time.

The Society met on Thursday afternoon in two sections, one for Analysis and Applied Mathematics and one for Geometry. Later in the afternoon at a meeting of the Society which was also a general session of the American Association for the Advancement of Science, Professor Vannevar Bush of the Massachusetts Institute of Technology delivered the twelfth Josiah

Willard Gibbs Lecture, entitled *Mechanical analysis*. Professor K. T. Compton, President of the American Association for the Advancement of Science, presided.

On Tuesday afternoon members of this Society and the Mathematical Association were guests of the Washington University chapter of Pi Mu Epsilon on a short sightseeing trip followed by a tea in the Women's Building of Washington University.

The banquet of the members of the Society and the Mathematical Association and their guests was held on Wednesday evening in the Coronado Hotel. Two hundred twenty-six persons attended this dinner. Professor R. D. Carmichael acted as toastmaster. He called on Professors W. H. Roever, W. C. Graustein, and E. R. Hedrick. The dinner was adjourned in time for members to attend a lecture by Professor R. C. Archibald on *Babylonian mathematics with special reference to recent discoveries*, which was given in connection with the dinner of the National Council of Teachers of Mathematics. A resolution of thanks was passed to the many people in St. Louis whose work had contributed to the success of the meeting.

The Board of Trustees held a meeting in the Coronado Hotel on December 31, 1935, and an adjourned meeting in the Faculty Club of Columbia University at 4:45 p.m. on January 4, 1936. The Council met in the Coronado Hotel at 7:30 p.m. on December 31, 1935. The Annual Business Meeting was held on Wednesday afternoon at 2:00 p.m. Two hundred eighteen ballots were cast and the following officers were elected:

Vice-Presidents, Professors E. B. Stouffer and Norbert Weiner.

Secretary, Dean R. G. D. Richardson.

Treasurer, Professor G. W. Mullins.

Associate Secretaries, Professors M. H. Ingraham and T. M. Putnam.

Librarian, Professor R. C. Archibald.

Member of the Editorial Committee of the Bulletin, Professor D. R. Curtiss.

Member of the Editorial Committee of the Transactions, Professor W. C. Graustein.

Member of the Editorial Board of the American Journal of Mathematics, Professor J. F. Ritt.

Member of the Editorial Committee of the Colloquium Publications, Professor Oswald Veblen.

Members of the Council, Professors Jesse Douglas, M. H. Stone, J. M. Thomas, T. Y. Thomas, R. M. Winger.

It was announced that the following eighty-seven persons had been elected to membership in the Society:

Mr. Lovincy Joseph Adams, Beverly Hills High School, Beverly Hills, California;
Mr. George Eugene Albert, University of Wisconsin;
Professor Hugh H. Barr, Harris Teachers College, St. Louis, Missouri;
Dr. Gertrude Blanch, Hunter College;
Miss Jessie W. Boyce, State Teachers College, Wayne, Nebraska;
Mr. Samuel Bronstein, Hartford Federal College, Hartford, Connecticut;
Dr. Newton Henry Brown, Arkansas State College, Jonesboro, Arkansas;
Miss Adelaide W. Bull, Essex County Junior College, Newark, New Jersey;
Professor Ezra John Camp, Yankton College;
Professor Loula Martha Caraher, Arkansas State Teachers College, Conway, Arkansas;
Professor William H. H. Cowles, Pratt Institute;
Professor Harry E. Crull, University of Southern California;
Professor Walter Thomas Daniels, Negro Agricultural and Technical College, Greensboro, North Carolina;
Professor Robert Dodds Daugherty, Kansas State College;
Mr. Uri P. Davis, University of Florida;
Dr. Thomas Luther Downs, Harvard University;
Professor Lyle Walker Finley, Monmouth College, Monmouth, Illinois;
Professor Bernice Colette Fites, Nazareth College, Nazareth, Michigan;
Mr. Irving Gordon Foster, University of Wisconsin;
Dr. Donald L. Fuller, American Cyanamid Company, Linden, New Jersey;
Brother Godfrey Vassallo, University of Portland, Portland, Oregon;
Mr. Julius Goldman, Detroit Institute of Technology;
Mr. Willard Lee Graves, University of Pennsylvania;
Miss Harriet M. Griffin, Brooklyn College;
Dr. Borghild Gunstad, University of Minnesota;
Mrs. Euphemia L. Haynes, Miner Teachers College, Washington, D. C.;
Professor Eugene W. Hellmich, Northern Illinois State Teachers College, DeKalb, Illinois;
Mrs. Susanne Edmondson Hotelling, New York City;
Mr. Joseph Jablonower, Fieldston School, New York City;
Mr. Robert Franklin Jackson, Harvard University;
Dean Paul Vernon Jewell, State Agricultural and Mechanical College, Orangeburg, South Carolina;
Dr. Nathan Kaplan, Brockton, Massachusetts;
Mr. Albert Arthur Koch, United States Engineers Office, Los Angeles;
Mr. Frank A. La Motte, Junior College, Centenary Collegiate Institute, Hackettstown, New Jersey;
Mrs. Lillian R. Lieber, Long Island University;
Professor Maceo Windfield Long, Samuel Huston College, Austin, Texas;

Dr. Arnold Noah Lowan, Yeshiva College, New York City;
Mr. Charles Anthony McCane, Miner Teachers College, Washington, D. C.;
Professor Willis Franklin Maclin, Mississippi Industrial College, Holly Springs, Mississippi;
Professor Frank P. Maguire, State Teachers College, East Stroudsburg, Pennsylvania;
Sister Maria Corona, College of Mount St. Joseph, Mount St. Joseph, Ohio;
Sister Mary Angela Margraf, Ursuline College, Cleveland, Ohio;
Sister Mary Resignata, Clarke College, Dubuque, Iowa;
Professor Julio A. Mira, College of the Sacred Heart, New York City;
Professor Sara Louise Nelson, Georgia State College for Women, Milledgeville, Georgia;
Mr. Edward Alfred Nordhaus, University of Wisconsin, Extension Center;
Mr. Hubert V. Park, State College of Agriculture and Engineering of the University of North Carolina;
Mr. Donald Theodore Perkins, Yale University;
Professor Charles Semple Pettis, Wofford College, Spartansburg, South Carolina;
Miss Theresa Louise Podmele, East High School, Buffalo, New York;
Professor J. W. Querry, Sam Houston State Teachers College, Huntsville, Texas;
Mr. K. Nagaraja Rao, Annamalai University, Annamalainagar, South India;
Dr. Louis M. Rauch, Berkeley, California;
Professor Bruce Hampton Redditt, Northeast Center, Louisiana State University, Monroe, Louisiana;
Professor Fred R. Reel, Lewis Institute, Chicago, Illinois;
Dr. Malcolm Slingsby Robertson, Yale University;
Dr. Willis Brainard Robinson, American International College;
Mr. Walter Roth, State University of Iowa;
Mr. John T. Rule, Taylor School for Boys, Clayton, Missouri;
Dean Henry Thomas Sampson, Jackson College, Jackson, Mississippi;
Mr. Max Sasuly, National Recovery Administration, Department of Commerce, Washington, D. C.;
Professor John Schon, St. Joseph's College, Collegeville, Indiana;
Professor Laurence W. Sheridan, St. Thomas College, Scranton, Pennsylvania;
Professor Victor Louis Sherman, Lewis Institute, Chicago, Illinois;
Professor Edwin Roscoe Sleight, Albion College, Albion, Michigan;
Mr. Charles Bassel Smith, University of Wisconsin;
Mr. Robert Edward Smith, Meadville, Pennsylvania;
Rev. Frederick Wyatt Sohon, Georgetown University;
Mr. Oswald S. Specht, Union College, Lincoln, Nebraska;
Professor Frederick H. Steen, Georgia School of Technology;
Professor Wren A. Stone, New Mexico Agricultural and Mechanical College, State College, New Mexico;
Professor Joseph Manual Synnerdahl, St. Francis Xavier College for Women, Chicago, Illinois:
Professor Walter Richard Talbot, Lincoln University, Jefferson City, Missouri;

Dean Marion B. Tolar, Fenn College, Cleveland, Ohio:
Professor Joseph C. Trainor, Washington State Normal School, Ellenburg, Washington;
Mr. Waldo Anthony Vezeau, St. Louis University;
Mr. John Ord Watts, Queen's University;
Professor Edward B. Wedel, Holdenville Junior College, Holdenville, Oklahoma;
Mr. Robert Lewis Westhafer, Hastings College, Hastings, Nebraska;
Professor Leslie Harper Whitcraft, Ball State Teachers College, Muncie, Indiana;
Miss Angeline Wilson, Grand Rapids Junior College, Grand Rapids, Michigan;
Professor William Woodhull Wood, Davidson College;
Mr. John William Wrench, Jr., Yale University;
Mr. John Franklin Wyckoff, South Dakota State College;
Dr. Clarence R. Wylie, Ohio State University;
Mr. Herbert Zuckerman, University of California.
As nominee of Brooklyn College: Mr. Vincent Dillon, Brooklyn College.

The secretary reported that the following persons had been admitted to membership in the Society in accordance with the reciprocity agreement with the London Mathematical Society:

Professor Edward Thomas Copson, University College, Dundee, Scotland;
Professor B. R. Seth, Hindu College, Delhi, India;
Professor S. M. Shah, Muslim University, Udebir Compound, Aligarh, India.

The following ten institutions and groups were elected to institutional membership in the Society:

Bowdoin College;
Department of Mathematics of Brooklyn College;
Connecticut College;
Dartmouth College;
Department of Mathematics of the University of Illinois;
University of Kansas;
Louisiana State University;
Pennsylvania State College;
Stevens Institute of Technology;
University of Virginia.

The Secretary reported that the membership in the Society is now 1863, including 117 contributing members, 75 nominees of institutional members and 79 life members. There are also 77 institutional members. The total attendance of members at all meetings in 1935 was 959; the number of papers read was 436; the number of members attending at least one meeting was 618.

The reports of the Treasurer and of the auditors (Professor

B. P. Gill and Mr. S. A. Joffe) showed a balance of $9,837.14 exclusive of the balances of the Bulletin, Transactions, Colloquium, Journal, Library, Sinking Fund, and special funds. The Society's Endowment Fund is invested in securities valued at $72,568.26 (as amortized in March, 1934). The income from various sources as listed in the Treasurer's report was:

Annual dues and initiation fees	$12,069.02
Sale of publications	11,620.24
Contributions toward publication	644.88
Institutional memberships	5,605.00
Individual contributing memberships	907.00
Rockefeller subvention	4,500.00
Special contributions	198.50
Interest from endowment	2,863.03
Other interest and miscellaneous	997.71
Total income for 1935	$39,405.38

The receipts from sales of the Society's publications does not include receipts from Bliss's *Algebraic Functions*, which have been transmitted to the Revolving Book Fund of the National Research Council, or income of special funds. The Trustees adopted a budget for 1936 showing estimated expenditures and receipts as $34,211.35 and $31,550.00, respectively. The indicated deficit, however, is largely fictitious as it is represented by outstanding bills which will be paid from the 1935 balances.

The Librarian reported that the Library of the Society now contains 8,612 volumes.

The following appointments were reported: as acting-treasurer until October 1, 1936, Professor P. A. Smith; as tellers for the election at the annual meeting, Professor Otto Dunkel and Dr. E. R. Lorch; as representative on the National Research Council for the period beginning July 1, 1936, to succeed Dean R. G. D. Richardson, Dr. T. C. Fry; as representatives on the Council of the American Association for the Advancement of Science for 1936, Professors M. H. Ingraham and C. N. Moore; as committee on arrangements for the 1936 annual meeting in Washington, D. C., Professors J. H. Taylor (chairman), F. E. Johnston, J. R. Kline, A. E. Landry, F. M. Weida, and Mr. W. D. Lambert; as the nominating committee, Pro-

fessors A. B. Coble (chairman), Marston Morse, M. H. Stone, E. B. Stouffer, and R. L. Wilder; as representative of the Society at the inauguration of Alan Valentine as president of the University of Rochester, November 15, 1935, Professor Virgil Snyder; as representative of the Society at the inauguration of John Ahlum Schaeffer as president of Franklin and Marshall College, December 6, 1935, Professor J. R. Kline; as additional members of the committee on arrangements for the 1935 annual meeting, Dr. Francis Regan and Dean A. S. Langsdorf; as representative of the Society at the inauguration of C. Valentine Boyer as president of the University of Oregon, February 6, 1936, Professor F. L. Griffin.

It was reported that Professor Jesse Douglas had accepted the invitation to give the Colloquium Lectures in 1937 and that his subject would be *The problem of Plateau*.

It was announced that the first volume of E. H. Moore's *General Analysis* has been published by the American Philosophical Society and that the University of Pennsylvania Press is acting as agent in its distribution.

Future meetings of the Society were announced as follows: Seattle, June 18; New York, October 31; Lawrence, Kansas, November 27–28.

The editors of the Transactions announced that the following persons had been asked to act as associate editors for 1936: Professors A. A. Albert, E. T. Bell, Jesse Douglas, T. H. Hildebrandt, Einar Hille, E. P. Lane, R. E. Langer, C. C. MacDuffee, Marston Morse, G. Y. Rainich, H. L. Rietz, M. H. Stone, J. L. Synge, G. T. Whyburn, and Oscar Zariski, and Dr. Caroline E. Seely (until July).

About three hundred persons attended the sessions other than the Gibbs Lecture. This attendance included the following two hundred thirty members:

V. W. Adkisson, E. S. Allen, C. B. Allendoerfer, N. B. Allison, N. L. Anderson, W. E. Anderson, R. C. Archibald, C. S. Atchison, H. T. Aude, R. W. Babcock, Wealthy Babcock, C. L. Bacon, H. W. Bailey, G. A. Baker, R. W. Barnard, I. A. Barnett, H. H. Barr, W. D. Baten, A. A. Bennett, Hermann Betz, William Betz, Solomon Bilinsky, F. L. Black, Archie Blake, Henry Blumberg, L. M. Blumenthal, Salomon Bochner, D. G. Bourgin, O. K. Bower, J. W. Boyce, J. W. Bradshaw, W. C. Brenke, R. W. Brink, F. L. Brooks, G. S. Bruton, R. S. Burington, J. Hughes Bushey, W. D. Cairns, B. H. Camp, C. C. Camp, R. D. Carmichael, H. C. Carter, E. W. Chittenden, A. B. Coble,

L. M. Coffin, L. W. Cohen, J. B. Coleman, J. T. Colpitts, E. G. H. Comfort, J. J. Corliss, Byron Cosby, N. A. Court, A. T. Craig, C. C. Craig, Paul Cramer, A. R. Crathorne, D. R. Curtiss, J. H. Curtiss, E. H. Cutler, R. D. Daugherty, H. T. Davis, Vincent Dillon, L. L. Dines, J. L. Doob, Otto Dunkel, P. D. Edwards, H. J. Ettlinger, H. S. Everett, G. M. Ewing, B. F. Finkel, C. H. Fischer, T. C. Fry, F. J. Gerst, J. L. Gibson, W. C. Graustein, F. L. Griffin, Marshall Hall, E. H. Hanson, E. G. Harrell, E. L. Harris, W. L. Hart, W. W. Hart, M. C. Hartley, M. L. Hartung, J. O. Hassler, T. W. Hatcher, E. R. Hedrick, C. B. Hennel, Fritz Herzog, D. M. Hickey, T. H. Hildebrandt, J. J. L. Hinrichsen, P. G. Hoel, M. A. Holly, H. M. Hosford, E. M. Hove, Mildred Hunt, W. A. Hurwitz, C. A. Hutchinson, Emma Hyde, M. H. Ingraham, Joseph Jablonower, Dunham Jackson, Nathan Jacobson, R. D. James, R. L. Jeffery, M. M. Johnson, L. C. Karpinski, D. E. Kearney, Claribel Kendall, G. S. Ketchum, P. W. Ketchum, E. W. King, S. C. Kleene, E. E. Knight, J. C. Knipp, Alfred Korzybski, J. H. Kusner, Cornelius Lanczos, C. G. Latimer, Solomon Lefschetz, Harry Levy, M. I. Logsdon, Dorothy McCoy, J. V. McKelvey, A. S. McMaster, C. C. MacDuffee, Saunders MacLane, V. S. Mallory, Morris Marden, Anna Marm, W. T. Martin, Mary N. Arnoldy, J. R. Mayor, C. W. Mendel, H. A. Meyer, H. J. Miles, W. E. Milne, W. L. Miser, U. G. Mitchell, E. C. Molina, Deane Montgomery, C. N. Moore E. J. Moulton, S. B. Myers, M. M. Ness, C. V. Newsom, G. D. Nichols, C. O. Oakley, Alta Odoms, Rufus Oldenburger, E. G. Olds, Arthur Ollivier, J. O. Osborn, W. V. Parker, W. O. Pennell, O. J. Peterson, H. B. Phillips, A. E. Pitcher, H. S. Pollard, J. W. Querry, T. C. Rafferty, J. F. Randolph, T. H. Rawles, Francis Regan, B. L. Remick, D. P. Richardson, R. G. D. Richardson, H. L. Rietz, R. F. Rinehart, W. C. Risselman, J. H. Roberts, Robin Robinson, S. L. Robinson, W. H. Roever, Rose M. Cook, A. E. Ross, M. F. Rosskopf, W. E. Roth, J. T. Rule, L. L. Runge, R. G. Sanger, M. G. Scherberg, I. J. Schoenberg, L. S. Shively, R. L. Short, C. H. Sisam, C. D. Smith, E. R. Smith, G. W. Smith, H. L. Smith, R. G. Smith, F. W. Sohon, I. S. Sokolnikoff, A. A. Stafford, G. W. Starcher, J. Y. Stephens, Guy Stevenson, R. W. Stokes, E. B. Stouffer, W. T. Stratton, J. L. Synge, Gabriel Szegö, J. D. Tamarkin, M. E. Taylor, E. L. Thorndike, E. W. Titt, W. J. Trjitzinsky, C. B. Tucker, H. W. Tyler, A. L. Underhill, J. H. Van Vleck, J. I. Vass, W. A. Vezeau, G. E. Wahlin, R. J. Walker, L. E. Ward, W. G. Warnock, J. H. Weaver, Warren Weaver, M. J. Weiss, W. D. A. Westfall, J. J. Wheeler, L. H. Whitcraft, H. S. White, W. M. Whyburn, K. P. Williams, F. L. Wren, Kathryn Wyant, M. M. Young.

Titles and cross references to abstracts of papers read at the regular sessions follow below. Mr. Sewell was introduced by Professor Walsh, Dr. Norris by Professor C. C. Craig, Dr. Downs by Professor Graustein, Mr. Larguier by Professor Regan, and Mr. Zuckerman by Dr. James. The joint paper by Professor Barnett and Dr. Mendel was read by Professor Barnett; that of Professor Morse and Mr. Pitcher, by Mr. Pitcher; that of Professor Montgomery and Dr. Zippin, by Pro-

fessor Montgomery. The paper by Dr. Hamilton was read by Professor Ettlinger.

The papers were read as follows: papers numbered 1 to 9 in the Section on Analysis Tuesday morning, Professor H. B. Phillips presiding; papers numbered 10 to 16 in the Section on Number Theory and Algebra Tuesday morning, Professor C. C. MacDuffee presiding; papers numbered 17 to 25 in the General Session on Wednesday morning, Professors Solomon Lefschetz and W. A. Hurwitz presiding; papers numbered 26 to 29 in the General Session on Wednesday afternoon, Professor E. B. Stouffer presiding; papers numbered 30 to 33 in the section for Statistics on Thursday morning, Professor H. L. Rietz presiding; papers numbered 34 to 40 in the Section for Analysis on Thursday morning, Professors Solomon Lefschetz and Henry Blumberg presiding; papers numbered 41 to 45 in the section for Analysis and Applied Mathematics on Thursday afternoon, Professor C. N. Moore presiding; papers numbered 46 to 57 in the section for Geometry on Thursday afternoon, Professor Solomon Lefschetz presiding. The papers whose abstract numbers are followed by the letter *t* were read by title.

1. *On a certain integral equation quadratic in the unknown function*, by Professor I. A. Barnett and Dr. C. W. Mendel. (Abstract No. 42-1-14.)

2. *A note on the degree of polynomial approximation*, by Dr. J. H. Curtiss. (Abstract No. 42-1-29.)

3. *Theory of linear differential equations containing a parameter*, by Professor W. J. Trjitzinsky. (Abstract No. 41-11-404.)

4. *On a problem of approximation associated with a periodic Stieltjes fraction*, by Professor W. C. Risselman. (Abstract No. 42-1-36.)

5. *Location of the level curves of Green's function*, by Mr. W. E. Sewell. (Abstract No. 42-1-11.)

6. *Invariants of closed extremals*, by Professor Marston Morse and Dr. A. E. Pitcher. (Abstract No. 42-1-69.)

7. *On the momentum problem for distribution functions in more than one dimension*, by Dr. E. K. Haviland. (Abstract No. 42-1-61-*t*.)

8. *A generalization of Markoff's theorem*, by Mr. W. E. Sewell. (Abstract No. 42-1-68-*t*.)

9. *Indeterminate expressions involving an arbitrary function and its derivatives*, by Professor J. F. Ritt. (Abstract No. 42-1-70-*t*.)

10. *Characteristic roots of direct products of n-way matrices*, by Dr. Rufus Oldenburger. (Abstract No. 42-1-52.)

11. *A note on null sequences*, by Mr. Marshall Hall. (Abstract No. 42-1-12.)

12. *On the class number of a quaternion algebra with a negative fundamental number*, by Professor C. G. Latimer. (Abstract No. 41-11-386.)

13. *The number of representations of an integer as a sum of twelve, sixteen, or twenty squares*, by Dr. R. D. James. (Abstract No. 42-1-22.)

14. *A construction of prime ideals in terms of absolute values of an algebraic field*, by Dr. Saunders MacLane. (Abstract No. 41-11-384.)

15. *On the matrices AB and BA*, by Professor W. E. Roth. (Abstract No. 42-1-73.)

16. *On the Hilbert-Netto theorem for algebraic differential equations*, by Professor H. W. Raudenbush, Jr. (Abstract No. 42-1-66-*t*.)

17. *The metric characterization of a certain class of spaces*, by Dr. L. M. Blumenthal. (Abstract No. 42-1-47.)

18. *Dual abelian groups*, by Dr. Leo Zippin. (Abstract No. 42-1-38-*t*.)

19. *Many-parameter groups in n-space*, by Professor Deane Montgomery and Dr. Leo Zippin. (Abstract No. 42-1-33.)

20. *On the groups associated with certain cyclic curves* (preliminary report), by Professor V. W. Adkisson. (Abstract No. 42-1-24.)

21. *Orthogonal polynomials in two variables: formal properties*, by Professor Dunham Jackson. (Abstract No. 42-1-18.)

22. *The equivalence of sequence integrals and non-absolutely convergent integrals*, by Professor R. L. Jeffery. (Abstract No. 42-1-54.)

23. *On restricted convergence and restricted summability of double series*, by Professor C. N. Moore. (Abstract No. 42-1-34.)

24. *A note on recursive functions*, by Dr. S. C. Kleene. (Abstract No. 42-1-74.)

25. *A Grace-Heawood theorem for the critical points of Green's function*, by Professor Morris Marden. (Abstract No. 42-1-72.)

26. *On semi-linear equations*, by Professor C. O. Oakley. (Abstract No. 42-1-27.)

27. *Critical sets for functions*, by Professor W. M. Whyburn. (Abstract No. 42-1-41.)

28. *Complex rational fractions*, by Professor L. R. Ford. (Abstract No. 42-1-49-*t*.)

29. *A new approximation method in solving linear differential equations with rational coefficients*, by Professor Cornelius Lanczos. (Abstract No. 42-1-32.)

30. *On certain distributions derived from the multinomial distribution*, by Dr. Solomon Kullback. (Abstract No. 41-11-385-*t*.)

31. *Convexity properties of generalized mean value functions*, by Dr. Nilan Norris. (Abstract No. 42-1-13.)

32. *The frequency distribution for the mean of n independent chance variables when each is subject to the law* $y_0x^{p-1}(1-x)^{q-1}$, by Professor W. D. Baten. (Abstract No. 42-1-56.)

33. *On the admissibility of time series*, by Professor Francis Regan. (Abstract No. 42-1-6.)

34. *Cauchy's problem for the non-linear hyperbolic differential equation in three independent variables*, by Dr. E. W. Titt. (Abstract No. 42-1-40.)

35. *Carathéodory linear measure and Vitali's theorem*, by Dr. J. F. Randolph. (Abstract No. 42-1-45.)

36. *On the symmetrical structure of general point sets*, by Professor Henry Blumberg. (Abstract No. 42-1-46.)

37. *Cauchy fields of ordinary differential equations*, by Professor H. J. Ettlinger. (Abstract No. 42-1-25.)

38. *Non-unique solutions of first order ordinary differential equations*, by Mr. O. H. Hamilton. (Abstract No. 42-1-26.)

39. *Summation of general Fourier expansions on closed Lie groups*, by Professor Salomon Bochner. (Abstract No. 42-1-75.)

40. *Extensionalization in mathematics, mathematical physics, and general education: general semantics*, by Count Alfred Korzybski. (Abstract No. 42-1-64.)

41. *On the zeros of successive derivatives of integral functions*, by Professor I. J. Schoenberg. (Abstract No. 42-1-37.)

42. *On the logarithmic mean of double Fourier series*, by Miss Alta Odoms. (Abstract No. 42-1-35.)

43. *Temperature distribution in a slab of two layers*, by Professor R. V. Churchill. (Abstract No. 42-1-16-*t*.)

44. *The problem of the sheet*, by Dr. D. G. Bourgin. (Abstract No. 42-1-43.)

45. *The dynamical theory of electrical commutator machinery* (preliminary report), by Professor J. L. Synge. (Abstract No. 42-1-62.)

46. *Isometries of closed 2-dimensional analytic Riemannian manifolds into themselves*, by Dr. S. B. Myers (National Research Fellow). (Abstract No. 42-1-59.)

47. *On the Cevian tetrahedron*, by Professor Nathan Altshiller-Court. (Abstract No. 42-1-23.)

48. *Asymptotic lines through a planar point of a surface and lines of curvature through an umbilic*, by Dr. T. L. Downs, Jr. (Abstract No. 42-1-31-*t*.)

49. *On the differential line-geometry of space associated with the linear conformal transformations of the dual sphere* (preliminary report), by Dr. Robin Robinson. (Abstract No. 42-1-58.)

50. *Cross-points of lines on a cubic surface*, by Professor H. S. White. (Abstract No. 42-1-71.)

51. *A new form of the four-vertex theorem*, by Professor W. C. Graustein. (Abstract No. 42-1-76.)

52. *Applicability with preservation of both curvatures*, by Professor W. C. Graustein. (Abstract No. 42-1-77.)

53. *The Schönemann-Eisenstein irreducibility criteria in terms of prime ideals*, by Dr. Saunders MacLane. (Abstract No. 41-11-383.)

54. *The maximal orders of generalized quaternion division algebras*, by Dr. Ralph Hull. (Abstract No. 42-1-1-*t*.)

55. *Sets of functions and their limit functions*, by Professor H. J. Ettlinger and Mr. O. H. Hamilton. (Abstract No. 42-1-2-*t*.)

56. *A note on the associative law in logical algebras*, by Professor H. B. Curry. (Abstract No. 42-1-4-*t*.)

57. *Differentiable functions defined in arbitrary subsets of euclidean space*, by Professor Hassler Whitney. (Abstract No. 42-1-5-*t*.)

58. *On the dynamics of commutator machines*, by Mr. W. H. Ingram. (Abstract No. 42-1-7-*t*.)

59. *On a method for evaluating the moments of a Bernoulli distribution*, by Mr. E. H. Larguier. (Abstract No. 42-1-8-*t*.)

60. *Note on a singular integral.* II, by Dr. E. P. Northrop. (Abstract No. 42-1-9-*t*.)

61. *A critique of the continuum*, by Professor A. H. Copeland (Guggenheim Fellow). (Abstract No. 42-1-10-*t*.)

62. *The minimum of a definite integral with respect to unilateral variations*, by Professor J. D. Mancill. (Abstract No. 42-1-17-*t*.)

63. *On the summability of Fourier series*, by Dr. W. C. Randels. (Abstract No. 42-1-19-*t*.)

64. *On universal Waring theorems for cubic functions* (preliminary report), by Mr. Alvin Sugar. (Abstract No. 42-1-20-*t*.)

65. *New results for the number g(n) in Waring's problem*, by Mr. H. S. Zuckerman. (Abstract No. 42-1-21-*t*.)

66. *A note on topological groups*, by Mr. Garrett Birkhoff. (Abstract No. 42-1-15-*t*.)

67. *Highly composite ideals*, by Professor R. G. Archibald. (Abstract No. 42-1-28-*t*.)

68. *The Jacobi series on an unrestricted lemniscate*, by Dr. J. H. Curtiss. (Abstract No. 42-1-30-*t*.)

69. *Collections filling a plane*, by Professor J. H. Roberts. (Abstract No. 42-1-39-*t*.)

70. *Sets which satisfy certain avoidability conditions*, by Professor R. L. Wilder. (Abstract No. 42-1-42-*t*.)

71. *Group velocity*, by Dr. D. G. Bourgin. (Abstract No. 42-1-44-*t*.)

72. *On some asymptotic formulas in the additive theory of numbers*, by Professor Leonard Carlitz. (Abstract No. 42-1-48-*t*.)

73. *Special regions of regularity of functions of several complex variables*, by Dr. W. T. Martin (National Research Fellow). (Abstract No. 42-1-50-*t*.)

74. *Concerning essential continua of condensation*, by Professor R. L. Moore. (Abstract No. 42-1-51-*t*.)

75. *Fundamental systems of units in normal fields*, by Professor M. J. Weiss. (Abstract No. 42-1-53-*t*.)

76. *On elementary number theory*, by Dr. Max Zorn. (Abstract No. 42-1-55-*t*.)

77. *On subanalytic spaces*. I, by Dr. Max Zorn. (Abstract No. 42-1-63-*t*.)

78. *Almost periodic properties of bounded solutions of linear differential equations with almost periodic coefficients*, by Dr. R. H. Cameron. (Abstract No. 42-1-57-*t*.)

79. *On singular Fourier-Stieltjes transforms*, by Mr. R. B. Kershner. (Abstract No. 42-1-60-*t*.)

M. H. INGRAHAM, *Associate Secretary*

RAYMOND JOSEPH GARVER—IN MEMORIAM

October 30, 1901–November 7, 1935

Friends and colleagues of Raymond J. Garver were grieved by his death on November 7, 1935. Professor Garver's untimely death in his early thirties interrupted a highly productive mathematical career and deprived his university of the services of one of its most useful and promising young men.

Professor Garver's work for the degree of Doctor of Philosophy at the University of Chicago was directed by Professor L. E. Dickson. His thesis on *Tschirnhaus transformations for general algebraic equations* appeared in Volume 28 (1927) of Annals of Mathematics. This work on the theory of equations inaugurated an extended series of articles and notes that he published in this field during the following eight years of his life. He also wrote a number of papers on *group postulates* and did considerable work on the mathematical problems of economics. An interesting and useful historical paper entitled *The Analyst* was published by him in Volume 1 (1933) of Scripta Mathematica. A complete list of his publications is to appear in an early issue of the National Mathematics Magazine.

Dr. Garver was a member of the American Mathematical Society and of the Mathematical Association of America. He was a regular contributor to the programs of both of these societies, especially when the meetings were held on the Pacific Coast. He was a member of the Editorial Board of the National Mathematics Magazine.

It is a very difficult task to form any kind of appraisal of the mathematical works of any person at the time of his death, even when that person has had what might be called a normal span of life. This becomes a practical impossibility when, as in the present case, the person died at the very threshold of his mathematical career. Professor Garver's earliest paper has been in print a scant eight years. If we may judge by the subjects that he investigated, the journals by which many of his articles were accepted and printed, and other things which appear when his works are examined, we may state with confidence that he made many substantial contributions to mathematics. His death at the peak of his career was a severe loss to mathematics and to the university that he served.

W. M. WHYBURN

JOHN IRWIN HUTCHINSON—IN MEMORIAM

On Sunday, December 1, 1935, his friends and colleagues were shocked to learn of the sudden and unexpected death of Professor J. I. Hutchinson, after forty-one years of service as a teacher of mathematics at Cornell University.

Born in Bangor, Maine, April 12, 1867, of typical New England stock, he graduated from Bates College in 1889 and the following year entered Clark University to pursue the study of mathematics under Professor Bolza. When Bolza was called to the newly organized University of Chicago, in 1892, Hutchinson followed him and was in the first group to obtain the doctorate from that institution. He accepted an instructorship at Cornell in 1894, was advanced to an assistant professorship in 1903, and to a full professorship in 1910. In 1896 Professor Hutchinson married Genevra Barrett, of Maine, who survives him. Their home was one of culture and refinement. Mr. Hutchinson was a skilled pianist. He had one of the finest flower gardens in Ithaca.

He very soon developed a comprehensive grasp of certain chapters of analysis, particularly of theta functions and of automorphic functions. His introduction of the isometric circle and its use in the determination of the fundamental region have had a profound influence in the subsequent development of the theory.

He also applied theta functions to the study of the general Kummer surface and showed that it is invariant under an infinite discontinuous group of birational transformations. This concept has been further developed by a number of later writers.

In 1912 Hutchinson had a nervous breakdown which interrupted his productivity for a time. When he recovered sufficiently to resume his activity his main interest was in the analytic theory of numbers, particularly the generalized zeta function. The results he achieved in this line are fully summarized in Titchmarsh's Cambridge Tract on that subject.

A long and active life has run its course, but its fruits still survive.

VIRGIL SNYDER

FOUR BOOKS ON GROUP THEORY AND QUANTUM MECHANICS

Gruppentheorie und Quantenmechanik. By Hermann Weyl. Second revised edition. Leipzig, S. Hirzel, 1931. xi+366 pp.

The Theory of Groups and Quantum Mechanics. By Hermann Weyl. Translated from the second revised German edition by H. P. Robertson. New York, E. P. Dutton, 1932. xxii+422 pp.

Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren. By Eugen Wigner. (Die Wissenschaft: Einzeldarstellungen aus der Naturwissenschaft und der Technik, Band 85, herausgegeben von Wilhelm Westphal.) Braunschweig, F. Vieweg und Sohn, 1931. viii+332 pp.

Die Gruppentheoretische Methode in der Quantenmechanik. By B. L. van der Waerden. (Die Grundlehren der Mathematischen Wissenschaften in Einzeldarstellungen, Band 36.) Berlin, Springer, 1932. viii+157 pp.

The theory of groups has always played a role in the formulation and technical exploitation of mathematical theories of natural phenomena; but this role has often been merely a subordinate or an implicit one. Thus, for example, the analysis of space and time has always involved group-theoretic considerations; and classical dynamics has tended to an ever closer relation with the theory of groups through the transformation theory and the various related devices which play so important a part in the studies of Poincaré and Birkhoff. Nevertheless, these branches of mathematical physics have been advanced on the classical level without any essential use of the more profound portions of the theory of groups. In the transition to the radically altered theories of space, time, and motion which are currently accepted, the role of the theory of groups has been notably enhanced. This is true of the relativity theory and, to an even greater degree, of the more recently formulated quantum dynamics. Indeed, there are extensive portions of the quantum-theoretical discussion of atoms and molecules which would be only partially intelligible without some knowledge of the theory of the representations of an abstract group by means of linear transformations. Since the new physical theories spring from radical attempts to eliminate untenable epistemological postulates from the analysis of natural phenomena, it is not altogether surprising that they require more varied and more subtle contributions from mathematics (regarded as the appropriate instrument of abstract rational speculation) in proportion as they couple a wider range of physical fact with a narrower a priori basis of intuitive judgment.

In reviewing any work on the quantum theory, one is tempted to essay a brief statement of the fundamental epistemological principles which determine in broad outline the general mathematical structure of the whole theory. In the present instance such a statement would prove singularly valuable in enabling us to indicate in a quite precise fashion the essential role of group theory and even to forecast advances which may be expected from certain extensions

of our present knowledge of group representations. While the task would not be a formidable one, it would carry us beyond the limits of a simple review and therefore cannot be undertaken here. Without having developed the necessary general background, we can do no more than consider, in somewhat vague terms, an important and instructive special case. For this purpose we select the atom. Familiar empirical evidence permits us to describe the atom as a compound dynamical system, built up of simpler interacting systems—the nucleus and a number of electrons—and characterized by certain properties of symmetry. The spatial symmetry about the nucleus and the essential likeness of the electrons are mathematically expressible in terms of the group of space rotations, $\mathfrak{d}_3$, and the group of permutations upon n objects, or symmetric group, π_n. Thus the theory of the groups $\mathfrak{d}_3$ and π_n must enter into the discussion of the atom, whatever may be the assumptions about the further properties of the constituent systems and their interactions. The physically observable wave characteristics of the electron, on the other hand, suggest the introduction of the mathematical apparatus for expressing undulatory phenomena as linear superpositions of elementary components. It is by virtue of this underlying linearity of wave theory that the linear representations of the groups $\mathfrak{d}_3$ and π_n have come to play so important a part in the quantum-theoretical treatment of the atom. Wigner was the first to observe this connection and to appreciate its significance as the essential key to the complex order of the atomic spectra. His fundamental paper appeared in the Zeitschrift für Physik (vol. 43 (1927), p. 624). The situation, however, is somewhat less simple than we have so far indicated. In order to arrive at a correct theory of the atom, it is necessary to make more precise the characteristics assigned to the electron. Before the development of the wave theory of the electron, Goudsmit and Uhlenbeck had already pointed out that the behavior of the electron could be brought into closer harmony with spectroscopic observations by ascribing to it an appropriate magnetic moment or spin. In a paper which appeared in the same volume of the Zeitschrift für Physik as Wigner's, on page 601, Pauli showed how the wave and spin properties of the electron could be combined. It is found that the resulting theory of the spinning electron involved the group $\mathfrak{u}_2$ of unimodular unitary transformations on two variables. Since the group $\mathfrak{u}_2$ is a double-valued representation of the group $\mathfrak{d}_3$, its representation theory includes that of $\mathfrak{d}_3$. The spinning electron may be introduced on the basis of relativistic arguments, as a natural consequence of the Lorentz-invariant wave equations of the electron discovered by Dirac (Proceedings of the Royal Society, (A), vol. 117 (1928), p. 610, and vol. 118 (1928), p. 351). We may remark in passing that, despite the successful union between relativity theory and quantum theory brought about in the case of the single electron, a complete reconciliation of the two theories has not yet been effected. The properties of the spinning electron are not alone sufficient to account for the behavior of the atom. Both chemical and spectroscopic evidence point to the stable arrangement of the electrons of an atom in shells about the nucleus. To describe the empirical data Pauli in 1925 introduced his famous exclusion principle. For the theory of the spinning electron it may be expressed in the form: no two electrons in an atom exist in the same state. Mathematically the

exclusion principle assumes a quite different form: the description of the atom as a compound wave must be antisymmetric in the constituent spinning electrons. In the latter form one sees that the group π_n plays the dominating role. The relation of the group π_n and the exclusion principle to the study of the formation of molecules from atoms was developed by Heitler, London, Weyl, and Rumer, who showed that the phenomenon of chemical valence can be fully discussed on the basis of general theory. Thus, we can indicate the importance of the groups $\mathfrak{d}_3$, π_n, and $\mathfrak{u}_2$ in developing the theoretical structure of the atom in harmony with the chemical and spectroscopic data; but we must at the same time point out that there is nothing in our remarks that really goes to the heart of the matter or that ascribes to group theory any function except that of a very useful mathematical device in this connection. That the relation of group theory to the foundations of quantum mechanics is far more profound than we have been able to show in this illustration, is the thesis brilliantly maintained by Weyl in a paper (Zeitschrift für Physik, vol. 46 (1927), p. 1) and, more fully, in the book which is under review.

If this review had been more promptly written, it would surely have reflected in the extravagance of its phraseology the eager, almost feverish, interest displayed by physicists in the newly useful subject of group theory. For Wigner's fundamental observation suddenly confronted physicists with an unexpected need for the more profound portions of group theory and stimulated them to an enthusiastic study of a branch of mathematics which they had been inclined to neglect; and it was some little time before they were able to approach the critical task of selecting those parts of group theory which were to remain indispensable for their technical ends. The intervening years have brought the selective process to a point of temporary and approximate equilibrium, a state of affairs to which Weyl already refers in the preface to his second edition when he alludes to the feeling of relief experienced by many physicists over the elimination of the "group-pest" from their science. This elimination has been accomplished by virtue of several different circumstances: in the first place, the physicist may confine his attention to the particular groups which are important for his purposes, and these are, chiefly, the groups $\mathfrak{d}_3$, π_n, $\mathfrak{u}_2$ noted above and the groups occurring in crystallography; in the second place, the linear representations of many of these groups can be determined in ways already familiar independently of any general theory, as is the case with the representations of $\mathfrak{d}_3$ in terms of the spherical harmonics; and, in the third place, Slater's observation that the Pauli principle can be applied in the preliminary stages of theoretical discussion simply by writing down the necessary antisymmetric wave function avoids most of the complications arising from the symmetric group π_n. While one can easily understand the physicist's preference for special mathematics as opposed to the general and the abstract, one cannot help feeling that the elimination is quite possibly undesirable and surely not definitive. An attentive reading of Weyl's book or a careful consideration of the basic concepts of the quantum theory give grounds for believing that portions of the representation theory not yet well understood by mathematicians may prove indispensable in the further development of mathematical physics. Certainly, if this be the case, no theoretical physicist will have cause for regret

if, by present attention to one of the most beautiful portions of abstract mathematics, he later finds himself prepared for one of those bewilderingly rapid advances which seem to be the lot of his subject.

Of the books before us, the most general in scope and the most profound in penetration is that of Weyl. As we have already remarked above, it represents the full and detailed elaboration of the ideas which he first set forth in a paper published in the Zeitschrift für Physik. Indeed, the paragraphs of this paper can be found scattered through the pages of the book. In main outline, the second edition now under review is like the first. There have been introduced many modifications and additions, enumerated in the foreword: in particular, much new material concerning the unification of the relativity and quantum theories has been incorporated in the second edition; and the difficult fifth chapter, which deals with the symmetric group and its application to the problem of valence, has been entirely revised. The scheme of presentation consists in the alternation of mathematical and physical material. The first chapter treats the geometry of unitary spaces, concluding with remarks on the generalization to Hilbert space (of which little or no use is made in the subsequent developments). The second chapter presents the general mathematical structure of quantum dynamics together with the perturbation methods and the discussion of many particular problems, such as the spectrum of the hydrogen atom and the interaction of the atom with radiation. Chapter 3 is devoted to the general theory of groups and their linear representations, with special attention to particular groups of interest for physical applications. Chapter 4 proceeds to apply the theory of groups to atomic spectra, the periodic table, the Lorentz-invariant wave equations of Dirac, and the Maxwell-Dirac field equations for quantum electrodynamics. Finally, the fifth chapter deals with the subject matter already noted above. The discussion of fundamental concepts and principles, whether mathematical or physical, is always stimulating and suggestive, conveying to the reader an extraordinarily vivid impression of the sweeping power of abstract mathematical thought. It must be said that the author's style, at once grandiloquent and concise, does not make easy the reading of the abstract material which he treats. To be sure, no abstract mathematical discussion can be fully appreciated without attentive study and some examination of concrete illustrations; but in the present instance the reviewer is persuaded that the general opinion regarding the difficulty of the book must be explained in part upon other grounds. Since this difficulty does not arise from ambiguity or lack of precision, the student who seriously desires to master this outstanding contribution to the mathematical literature of the quantum theory should not be deterred from doing so.

The translation of Weyl's book by Robertson was undoubtedly intended to render the great wealth of material which it contains more readily accessible to the student whose mother tongue is English. The reviewer's comments on the difficulty of reading the book now help to indicate the special value of the translator's service to English and American students; for many his work will help to reduce obstacles which, if not insurmountable, are at least very serious when met in a foreign language. The translation is essentially a literal one. It has been done painstakingly and well, in spite of a few confusing errors which have been brought to the reviewer's attention. Since some of these errors are

not merely typographical in nature, we shall cite them here for the possible benefit of users to whose attention this review may come. On page 118, the sentence beginning on line 13 should read, "In this way, the elements of $\mathfrak{g}$ are distributed into sets of elements mutually equivalent with respect to $\mathfrak{h}$." On page 154, the sentence beginning on line 10 should read: "But in any case we can find a definite column index h with the following properties: there exist non-vanishing L-matrices whose first $h-1$ columns vanish; but all L-matrices whose first h columns vanish satisfy $L=0$." On page 156, the sentence beginning on line 9 from the bottom should read: "$\mathfrak{h}$, $\mathfrak{h}'$ being irreducible representations . . . ." It should be noted that the somewhat varied typographical problems have been well handled by the printer.

Wigner's book is much more elementary in character than that of Weyl. The development is clear and as concrete as the subject permits. Many simple illustrations are given so that the reader may test the content of the more abstract portions. The first three chapters are devoted to the theory of matrices. They are followed by three chapters concerning the fundamental concepts of quantum mechanics, together with perturbation methods. The discussion of groups and their linear representations, with special reference to those of primary interest to the physicist, occupies Chapters 7–16. The application of the general theory to the study of atomic spectra is given in the next three chapters, without the introduction of the spinning electron. The discussion of spin is carried through in Chapter 20, which serves as a foundation for the four last chapters on atomic spectra. While in some places a mathematician would express himself differently or even a little more precisely, Wigner has provided here one of the best introductory accounts of the subject. From the physicist's point of view it probably has the advantage of performing an inoculation with the germs of the "group-pest" in approximately the correct quantity. The book is less stimulating then Weyl's; and it does not attempt to cover so wide a range of subject matter, the relativity and valence problems being excluded. Of course, it should be recalled that the discussion of valence by group-theoretical methods was in its infancy at the time the book was published. On the other hand, Dirac's wave equations were as well known at that time as they are today. The author's omission of the Dirac theory can therefore be regarded as an exercise of his own judgment. In consequence of his decision in this matter, he has been compelled to give an original treatment of the spinning electron, based on Newtonian relativity alone, which the reviewer considers one of the most interesting and suggestive parts of the book.

The book of van der Waerden seems to be confined so far as possible to the exposition of group-theoretic technique. The physical discussions are mainly brief summaries of the necessary background material without emphasis on general concepts, or straightforward applications of the facts of group theory to physical problems. The six chapters deal with an introduction to quantum mechanics; groups and their representations, with special treatment of the rotation and Lorentz groups and discussion of simpler cases by way of illustration, followed by applications to atomic spectra; Dirac's equations and the spinning electron, with a more complete discussion of spectra; the permutation group and the Pauli principle; molecular spectra and the formation of molecules. The exposition is extraordinarily clear and compact. In particular, the

treatment of group theory displays to the best advantage the simplification in technique obtainable through a proper use of abstraction. To the reviewer it seems that the book would be especially useful as a source and reference book in connection with discursive lectures. It could hardly be used as an introduction to the physical theory. While it does not contain an index, differing in this respect from all the other books under review, it is so brief that the table of contents is a quite effective guide to the specific subjects treated in the text.

For the student of the quantum theory, these books are indispensable if he wishes to master and to use the group-theoretical techniques appropriate to his problems. He will naturally wish to keep them on his shelves beside such well known theoretical works as Dirac's *Principles of Quantum Mechanics* and von Neumann's *Mathematische Grundlagen der Quantenmechanik*, in which the role of group theory is not emphasized. Mathematicians who value the fructifying contacts of their abstract realm with the more concrete world of physics will find this latest meeting both fascinating and inspiring; and those who interest themselves rather in the pure theory of groups will wish to consult the books of Weyl and van der Waerden both for their masterly exposition of the representation theory and for their suggestions of problems yet unsolved.

M. H. STONE

A CORRECTION

A regretted slip was made in my review of Smith and Ginsburg's *History of Mathematics in America* (this Bulletin, vol. 41 (1935), pp. 603–606), in the list of suggestions for a new edition. Line 16, page 606—namely, p. 37, l. 14, *for* "of" *read* "on"—should be eliminated.

R. C. ARCHIBALD

SHORTER NOTICES

An Introduction to the Theory of Functions of a Complex Variable. By E. T. Copson. Oxford, Clarendon Press, 1935. v+448 pp.

Until a few years ago Whittaker and Watson's *Modern Analysis* was practically the only modern British text on the theory of functions of a complex variable. Dienes' treatise on *The Taylor Series* appeared in 1931, Titchmarsh's *The Theory of Functions* in 1932. To these books Professor E. T. Copson of University College, Dundee, in the University of St. Andrews, has added a worthy companion. The book under review stands in some respects fairly close to the Whittaker and Watson model. In particular, it gives ample treatment of the special analytic functions, but with much less bewildering detail. It is more elementary than Titchmarsh's treatise, and does not devote any space to functions of a real variable.

A list of the chapter headings will give some idea of the actual contents: Complex numbers; The convergence of infinite series; Functions of a complex variable; Cauchy's theorem; Uniform convergence; The calculus of residues; Integral functions; Conformal representation; The gamma function; The hypergeometric functions; Legendre functions; Bessel functions; The elliptic functions of Weierstrass; Jacobi's elliptic functions; Elliptic modular functions and Picard's theorem.

The stated purpose of the book is to provide an easy introduction to the methods of the theory of functions of a complex variable, and it seems to be admirably suited for this purpose. The presentation is clear and sufficiently detailed. Difficulties are frequently avoided by restriction of the generality, but the author takes care to call the reader's attention to the existence of more general results and to the place where they can be found in the literature. There are plenty of references in the text as well as suggestions for collateral reading at the end of each chapter. There is also a wealth of interesting and important material tucked away among the many sets of examples.

The typography and make-up of the book are excellent, only the price is repellent.

EINAR HILLE

NOTES

In addition to the members of the Editorial Board of the Transactions of the American Mathematical Society, the following persons have assisted the editors by refereeing papers offered for publication in the volumes for 1935: C. R. Adams, A. A. Albert, L. E. Dickson, Nelson Dunford, J. J. Gergen, L. M. Graves, T. H. Hildebrandt, T. R. Hollcroft, Solomon Lefschetz, N. Levinson, Mayme I. Logsdon, C. C. MacDuffee, Wladimir Seidel, P. A. Smith, Virgil Snyder, Otto Szász, Morgan Ward, H. M. Westergaard, Norbert Wiener.

Professor A. Ferguson has been appointed president of Section A (Mathematical and Physical Sciences) of the British Association for the Advancement of Science.

A commission of the Mathematical Association of America has recently published a report on *The training of teachers of mathematics*. The report is concerned primarily with the preparation of teachers in colleges and universities, but includes a short discussion of the training of teachers for secondary schools.

The State University of Kief is publishing a new journal, Bulletin Scientifique, Recueil Mathématique. The first number appeared recently.

The Paris Academy of Sciences announces the award of the following prizes for 1935: the Francoeur prize to André Weil, for his work on algebraic bodies; the Montyon prize in mechanics to Pierre Dupin, for his work on the mechanics of fluids; the Poncelet prize in mechanics to Auguste Lafay, for his work in mechanics as a whole; the Gaston Planté prize in physics to Marcel Pauthenier, for his work in electricity; the Hébert prize in physics to Jean Fallou, for his *Les réseaux de transmission d'énergie*, in particular; an Henri de Parville prize in physics to Jean Mercier, for his work on electrical oscillations; the Hughes prize in physics to Louis Néel, for his work on magnetism; the Pierson-Perrin prize in physics to Pierre Auger, for his work in physics as a whole; an award from the Clément Felix Foundation to Charles Dietsch to continue his research on the measure of currents; the Montyon prize in statistics to Robert Gibrat, for his work on mathematical statistics; an Henri de Parville prize to Jean Villey, for his publications on applied thermodynamics; an Henri de Parville prize to Georges Darmois, for his *Statistique et applications*; the Binoux prize to George Sarton, for his *Introduction à l'histoire des sciences depuis l'antiquité jusqu'à Roger Bacon*; the Bordin prize to Henri Cartan, for his work on mathematical analysis; a Petit d'Ormoy prize of the mathematical sciences to Maurice Fréchet, for his mathematical work as a whole; a prize from the Hirn Foundation to Georges Giraud, for his work in theory of equations; a prize from the Madame Victor Noury Foundation to Jean Delsarte, for his work on mathematical analysis.

According to a statement recently issued by Colorado College, this college has become, during recent years, increasingly a center at which visiting research workers have been carrying on their investigations during the summer months. To encourage this movement, the college is prepared to offer during the summer free room accommodations to qualified investigators. Further information may be obtained from the members of the various departments or from the president of the college.

The Royal Society of London has recently awarded a Hughes Medal to Dr. C. J. Davisson, for his work on electrons, and a Royal Medal to Professor C. G. Darwin, Tait professor of natural philosophy at the University of Edinburgh, for his researches in mathematical physics.

Dr. A. E. Kennelly, professor emeritus at Harvard University and the Massachusetts Institute of Technology, has been awarded the 1936 Mascart Medal of the Société Française des Électriciens, Paris. The medal is awarded triennially for distinguished service in either basic or applied electricity.

Professor Roger Adams, head of the department of chemistry in the University of Illinois, has been awarded the Josiah Willard Gibbs Medal of the Chicago Section of the American Chemical Society.

At the annual meeting of the American Society of Mechanical Engineers Professor Stephen Timoshenko, of the University of Michigan, was awarded the Worcester Reed Warner Medal.

Professor Umberto Crudeli, of the Royal University of Palermo, has been appointed professor of mathematical physics at the Royal University of Naples.

Dr. Walter Rosemann, of the Technischen Hochschule Hannover, has been promoted to a professorship in mathematics.

Dr. F. K. Schmid, of the University of Erlangen, has been appointed to a professorship at the University of Jena.

Professor Emanuel Sperner, of Königsberg, will succeed Professor Konrad Knopp, of Tübingen, as editor of the Jahresberichte der deutschen Mathematiker-Vereinigung beginning with volume 46 (1936).

Professor Lars V. Ahlfors, of the University of Helsingfors, who has held a lectureship during the current academic year, has been appointed to an assistant professorship at Harvard University.

Professor R. C. Archibald, of Brown University, is on leave of absence the second semester of this academic year.

Mr. Garrett Birkhoff, of the Society of Fellows, has been appointed to an instructorship at Harvard University.

Professor G. D. Birkhoff has been appointed Dean of the Faculty of Arts and Sciences at Harvard University for three years beginning September 1, 1936. Professor Birkhoff is serving as acting Dean during the current academic year.

Assistant Professor J. Hobart Bushey, of Hunter College, has been promoted to an associate professorship in mathematics.

Professor Rudolph Carnap, of the University of Prague, is teaching at the University of Chicago this year. He is an authority on philosophy of science.

Assistant Professor L. W. Cohen, of the University of Kentucky, has been promoted to an associate professorship.

Associate Professor N. A. Court, of the University of Oklahoma, has been promoted to a professorship.

Associate Professor H. T. Davis, of Indiana University, has been promoted to a professorship.

Assistant Professor Edward Fleisher, of Brooklyn College, has been promoted to an associate professorship.

Professor W. W. Hart, of the University of Wisconsin, has retired. He is planning to write some texts for courses in mathematics in the secondary schools.

Dr. P. G. Hoel, of Rose Polytechnic Institute, has been promoted to an assistant professorship.

Assistant Professor O. W. Irvin, of Brooklyn College, has been promoted to an associate professorship in mathematics.

Dr. L. S. Kennison, of Brooklyn College, has been promoted to an assistant professorship.

Dr. Max Mason, who joined the Rockefeller Foundation as director for natural sciences in 1928, has announced his retirement. He is planning to resume his mathematical work.

Professor G. M. Merriman, of the University of Cincinnati, is on leave of absence for the second semester of this academic year and is at Harvard University.

Assistant Professor L. T. Moore, of Brooklyn College, has been promoted to an associate professorship.

Professor J. R. Musselman is on leave of absence from Western Reserve University for the second semester of the current year. He will spend some weeks at the Institute for Advanced Study, following this with travel in Belgium, Denmark, and Norway, including the International Congress of Mathematicians at Oslo.

Dr. N. E. Rutt, of Northwestern University, has been promoted to an assistant professorship.

Dr. Frank E. Smith, of Brooklyn College, has been promoted to an assistant professorship in mathematics.

Dr. R. C. Stephens, of Knox College, has been promoted to an assistant professorship.

The following appointments to instructorships are announced: Brooklyn College, Miss Mary Draper; Kansas State Teacher's College, Dr. C. B. Tucker; University of New Mexico, Mr. H. D. Larsen; St. Louis University, Dr. A. E. Ross.

Professor J. E. A. Steggall, emeritus professor of mathematics at University College, Dundee, in the University of St. Andrews, died on November 26, 1935, at the age of eighty years.

Assistant Professor Charles F. Bowles of the South Dakota School of Mines died on February 7, 1936, after an illness of several weeks.

Professor W. E. Byerly, emeritus professor of mathematics at Harvard University, died on December 20, 1935, at the age of eighty-six years.

Professor B. H. Crenshaw, of the Alabama Polytechnic Institute at Auburn, died on November 25, 1935, at the age of sixty-eight years.

Miss Julia Dale, of Duke University, died on January 15, 1936.

Professor A. B. Dinwiddie, president of Tulane University and professor of applied mathematics and astronomy there, died on November 21, 1935, at the age of sixty-four years.

Dean P. J. McHugh, of Boston College, died on January 8, 1935.

Dr. Henry Taber, professor of mathematics at Clark University from 1888 to 1921, and emeritus professor since 1921, died January 6, 1936, at the age of seventy-five years.

Mr. A. A. Welch, president of the Phoenix Mutual Life Insurance Company, died on May 8, 1935.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

82. Dr. Reinhold Baer: *The subgroup of the elements of finite order of an abelian group.*

Denote by $F(A)$ the subgroup of all the elements of finite order of the abelian group A. Then, if F is an abelian group whose elements are of finite order and J an abelian group without elements $\neq 0$ of finite order, the direct sum $A=F+J$ satisfies: $F(A)\backsim F$, $A/F(A)\backsim J$. $F+J$ is essentially the only group satisfying these conditions if, and only if, $F(A)$ is a direct summand of every abelian group A such that $F(A)\backsim F$, $A/F(A)\backsim J$. The pairs of groups F, J with this property are characterized by a simple relation between the structure of F and the structure of J. (Received January 24, 1936.)

83. Professor Clifford Bell: *On a theorem in higher plane curves.*

In Hilton's *Higher Plane Curves* the function $F(t)=|f(t)\phi'(t)\psi''(t)|$ appears in a theorem concerning the cusps and inflections of a plane curve which is represented parametrically by the equation $x:y:z=f(t):\phi(t):\psi(t)$. The theorem states that if f, ϕ, ψ satisfy certain conditions, the cusps are given by the repeated roots of $F(t)=0$, while the inflections are given by the single roots. In this note, making use of the Plücker numbers, a proof of the theorem is given which leads to additional information about the repeated roots. (Received January 6, 1936).

84. Professor Salomon Bochner and Dr. W. T. Martin (National Research Fellow): *Singularities of composite functions in several variables.*

A generalization to functions in several variables is formulated for the classical theorem of Hadamard on the composition of singularities and stars of regularity. The generalization involves an alternative restatement of the classical theorem itself. (Received January 20, 1936.)

85. Professor A. D. Campbell: *A note on primitive roots in a Galois field.*

By using the condition that a primitive irreducible n-ic modulo p of the form $X^n-A_1X^{n-1}-A_2X^{n-2}-\cdots-A_{n-1}X-A_n=0$ must not divide X^m-1

$\equiv 0$ for $m < p^n - 1$ the author derives a useful method for determining primitive roots of a Galois field of order p^n. (Received January 31, 1936.)

86. Professor Leonard Carlitz: *On higher congruences.*

This paper is concerned with congruences of the form $A_0 t^{p^{nm}} + A_1 t^{p^{n(m-1)}} + \cdots + A_m t \equiv B \pmod P$, where A, B, P are polynomials in an indeterminate x with coefficients in a Galois field of order p^n. For earlier results see abstract 41-9-336; also this Bulletin, vol. 41 (1935), pp. 907–914. In the present paper the homogeneous case ($B \equiv 0$) is assumed solved; criteria are then obtained for the solvability of the non-homogeneous case ($B \not\equiv 0$). The paper also contains some results on the homogeneous case as well as a new and simple proof of a criterion for the solvability of $t^{p^n} - t \equiv B \pmod P$. (Received January 24, 1936.)

87. Professor Leonard Carlitz: *On some arithmetic functions of several arguments.*

In this note are defined certain generalizations of the Möbius μ- and the Euler ϕ-functions. As an example, in the case of three variables, $\phi'''(m_1, m_2, m_3)$ is the number of triplets (u_1, u_2, u_3), $u_i \pmod{m_i}$, and $(u_1, u_2, u_3, m_1, m_2, m_3) = 1$; $\phi''(m_1, m_2, m_3)$ is the number of triplets for which $(u_1, u_2, m_1, m_2) = 1$ and two similar conditions hold. The functions were suggested by manipulating the series $\Sigma A(m_1, m_2, m_3)$ summed first over all m for which $(m_1, m_2, m_3) = 1$, and secondly over all m for which $(m_2, m_3) = (m_3, m_1) = (m_1, m_2) = 1$. (Received January 24, 1936.)

88. Professor Leonard Carlitz: *Some problems in additive arithmetic.*

This paper is concerned with sums of the type $\Sigma\alpha(m_1, \cdots, m_k)$ extended over all positive integral m_j for which $m = m_1 + \cdots + m_k$; α is an arithmetic function of $k \geqq 2$ arguments and satisfies certain order conditions. In an earlier paper (Quarterly Journal of Mathematics, vol. 3 (1932), pp. 273–290) the case $\alpha(m_1, \cdots, m_k) \equiv \alpha(m_1) \cdots \alpha(m_k)$ was treated. Similar results are now obtained for general $\alpha(m_1, \cdots, m_k)$; the proof is no longer by induction but otherwise very much like the earlier proof. The results are applied to the sum $\Sigma\alpha(m_1, \cdots, m_k)$ extended over all positive m_j and primes p_i such that $n = m_1 + \cdots + m_k + p_1 + \cdots + p_s$. (Received January 24, 1936.)

89. Dr. J. A. Clarkson (National Research Fellow): *Uniformly convex spaces.*

Banach spaces are considered in which the norm is uniformly convex; expressed in geometrical language, the mid-point of a variable chord of the unit sphere in the space cannot approach the surface of the sphere unless the length of the chord goes to zero. All finite dimensional euclidean spaces, and Hilbert space, possess this property: it is shown that all spaces L_p and l_p are uniformly convex if p exceeds unity. A strengthened form of the ordinary triangular inequality is derived for such spaces. By means of this relation it is proved that functions of bounded variation from a euclidean space into a

uniformly convex space are differentiable almost everywhere, that if such functions are absolutely continuous they are (Bochner) integrals of their derivatives, that a continuous rectifiable curve in a uniformly convex space has a tangent almost everywhere. (Received January 27, 1936.)

90. Mr. Nelson Dunford: *Integration of abstract functions.*

A sequence $f_n(P)$ of functions summable (in Bochner's or in the author's sense) on (0, 1) to the Banach space X may converge to $f(P)$ almost everywhere and one may have $\lim_n \int_e f_n(P)dP$ existing for every measurable set e without having $f(P)$ summable. An extended class of summable functions and an extended integral may be defined by putting $\int_e f(P)dP = \lim_n \int_e f_n(P)dP$. The integral is absolutely continuous and completely additive. The space of summable functions can be normed so as to be complete, and, in case X has a base, every function summable in the sense of Garrett Birkhoff is also summable in the extended sense to the same value. Every additive set function with values in a Banach space with a base which vanishes on sets of measure zero is absolutely continuous. (Received January 31, 1936.)

91. Mr. Nelson Dunford: *Linear operations.*

The general linear operator on L to $L_q(q>1)$ is given by the formula $T\phi = \int_0^1 K(P, Q)\phi(P)dP$ where the kernel satisfies the condition ess. $\sup_P \{\int_0^1 |K(P, Q)|^q dQ\}^{1/q} < \infty$. This number is the evaluation of $|T|$. T is completely continuous if and only if it satisfies the further condition $\lim_{h\to 0}$ ess. $\sup_P \int_0^1 |K(P, Q+h) - K(P, Q)|^q dQ = 0$. Analogous results hold with L replaced by the space of absolutely continuous functions, or with L_q replaced by a Banach space with a uniformly convex norm. If $f(P)$ on (0, 1) to $L_q(q \geqq 1)$ is measurable and if $\int_0^1 \|f(P)\|^{p/(p-1)} dP < \infty$, then the transformation $T\phi = \int_0^1 f(P)\phi(P)dP$ is completely continuous on L_p to L_q. This gives the author's (or Bochner's) integral a number of further properties in common with the Lebesgue integral of real functions. (Received January 31, 1936.)

92. Professor L. A. Dye: *Involutorial transformations associated with a ruled quartic surface with a double twisted cubic.*

A (1, 1) correspondence is established between the generators of a ruled quartic surface R which has a double twisted cubic and a simple line as directrices, and the surfaces of a pencil of quadrics. A general point P in space fixes a quadric and a generator which with P determines a plane tangent to R at a point Q. The line PQ meets the quadric in a residual point P' which is the image of P in involutorial transformation. Two cases are considered, one in which the simple directrix line is a part of the base of the pencil of quadrics, and in the other the twisted cubic lies on all surfaces of the pencil. The transformations are of order 29 and have some involved contact relations among the fundamental elements. (Received February 1, 1936.)

93. Professor H. T. Engstrom: *On substitution polynomials.*

A product of two polynomials $f(x)$ and $g(x)$ with coefficients in a field K is defined as $f(g(x))$. $F(x)$ is said to be irreducible if $F(x) \neq f(g(x))$ with $f(x)$

and $g(x)$ of degrees greater than 1. In decompositions of a polynomial into irreducible factors the number of factors is unique and their degrees are unique except for order. This theorem was proved by Ritt when K is the field of complex numbers. The author gives a purely algebraic proof which is valid for all fields K of characteristic 0. (Received February 1, 1936.)

94. Professor H. T. Engstrom and Dr. Max Zorn: *Geometry in a space-time of Page.* Preliminary report.

The authors show that the axioms of Page, which can be stated in essentially topological form, imply a definite structure for his "3-dimensional manifold of equivalent particle observers." The following results are obtained: 1. A description by coordinates; 2. The existence of an essentially unique time; 3. A metric admitting a transitive group of isometric mappings; 4. Ordinary spherical geometry. (Received February 1, 1936.)

95. Mr. Aaron Fialkow: *The geometry of single-parameter families of plane curves.*

Consider the single-parameter family of plane curves given by the equation $u(x, y)=$constant. Draw the orthogonal trajectories of these curves and study the geometry of the resulting orthogonal net under the assumption that $u(x, y)$ is a solution of one of the differential equations: (1) $u_{xx}+u_{yy}=f(u)$, (2) $u_{xx}u_{yy}-u_{xy}^2=f(u)$, (3) $u_{xx}u_y^2-2u_{xy}u_xu_y+u_{yy}u_x^2=f(u)$, (4) $u_{xx}(1+u_y^2)-2u_{xy}u_xu_y+u_{yy}(1+u_x^2)=0$. Through an arbitrary point of the plane there pass two perpendicular curves. At this point the curvature as well as the tangential and normal derivatives of the curvature are defined for each curve. The geometric properties of the net may be stated as equations in these intrinsic geometric quantities which are satisfied at each point. Complete geometric characterizations of this type for the four differential equations given above are obtained. There are also derived further conditions that $f(u)$ shall be a constant or zero in the first two equations. Some of these results may be interpreted as stating the necessary and sufficient conditions that a single-parameter family of curves be the projections of the ∞^1 parallel plane sections of a developable or minimal surface. (Received January 20, 1936.)

96. Dr. M. M. Flood: *The resultant matrix of two polynomials.*

If A is the square matrix of order n whose first row has the element $-a_k$ in the kth column, unity in the jth row and $(j-1)$st column for $j=2, 3, \cdots, n$, and zeros elsewhere, then the polynomial $f(x)=x^n+a_1x^{n-1}+\cdots+a_n$ is the characteristic function of A. If $g(x)$ is any other polynomial, the author calls $g(A)$ the *resultant matrix* of $g(x)$ and $f(x)$. Now if p is the degree of the HCF of $f(x)$ and $g(x)$, and if M_k, for $k=0, 1, \cdots, p$, denote the minors formed from $g(A)$ by using the last $n-p$ rows and first $n-p-1$ and $(n-p+k)$th columns, the principal theorem of this paper states that $M_0[g(x), f(x)]=M_0x^p+M_1x^{p-1}+\cdots+M_p$. This extends the theorem of W. V. Parker (American Mathematical Monthly, vol. 42 (1935), p. 164) which states only that the nullity of

the resultant matrix of two polynomials is equal to the degree of their HCF. (Received January 18, 1936.)

97. Mr. R. H. Fox: *On the asymptotic distribution of analytic almost periodic functions.*

In this paper is discussed the asymptotic distribution function of an analytic almost periodic or related function $f(s)$ in a strip $\alpha \leqq \sigma \leqq \beta$ of the s-plane where $s=\sigma+it$. It is shown that $f(s)$ possesses an asymptotic distribution function in a strip $\alpha \leqq \sigma \leqq \beta$ if, for example, $f(s)$ is uniformly almost periodic there. In addition, a simple integral formula connecting the strip distribution function with the linear series of line distribution functions belonging to the σ of the interval $\alpha \leqq \sigma \leqq \beta$ is developed, as well as an analogous relation connecting their Fourier transforms. The results are then applied to the case of independent moduli. It is found, in particular, that if $\alpha > 1/2$ the logarithm of the Riemann-zeta function possesses a strip distribution function which is absolutely continuous with a density having continuous partial derivatives of all orders; all moments exist and belong to a determined moment problem. If, in addition, $\beta < 1$, the density is a transcendental entire function of its two variables. (Received January 29, 1936.)

98. Professor L. M. Graves: *Under-determined systems of linear differential equations.*

It has been pointed out by Carrus that the general solution of a system of ordinary linear differential equations may usually be obtained by explicit formulas not even involving quadratures, provided the number of equations is fewer than the number of unknown functions. When the system of equations is written as a system of first order equations, the method and results can be stated much more clearly and briefly, as well as more completely than was done by Carrus. It is shown also that the method is applicable to certain equations in which the operation of differentiation is replaced by much more general additive operators. Certain types of linear partial differential equations of the first and second orders are discussed as examples. (Received January 29, 1936.)

99. Mr. D. W. Hall: *On the non-alternating images of linear graphs.*

In this paper a study is made of the possible images of a linear graph under a non-alternating transformation (see G. T. Whyburn, American Journal of Mathematics, vol. 46 (1934), no. 2). It is shown that: I. A necessary and sufficient condition that a cyclic curve C be the non-alternating image of a linear graph is that C be the sum of a finite number of simple arcs; II. If B be be the non-alternating image of a linear graph A, then every true cyclic element of B is the sum of a finite number of simple arcs. (Received January 29, 1936.)

100. Dr. E. K. Haviland: *On an asymptotic expression for a certain integral.*

An exact proof is given for the asymptotic formula $J=\int_0^\infty \exp\left[\alpha x-\phi(x)\right]dx$

$\sim\exp\ [\xi\phi'(\xi)-\phi(\xi)](2\pi/\phi''(\xi))^{1/2}$, where $\alpha=\phi'(\xi)$, under the conditions: (i) ϕ, ϕ', and ϕ'' are positive, continuous, and monotone increasing for non-negative x, and all three tend to infinity with x; (ii) ϕ''' exists and is positive if x is positive; (iii) ϕ'' and ϕ''' are such that ϕ'''/ϕ'' and ψ/ϕ'' tend to definite limits (including $\pm\infty$) as x becomes infinite, where ψ is a logarithmico-exponential function. It is shown by an example that if condition (iii) is not satisfied, the asymptotic formula need not hold. (Received January 16, 1936.)

101. Professor Edward Kasner: *A triangular inequality in conformal geometry.*

If three curves C_1, C_2, C_3 touch each other at one point, there are three horn angles C_1C_2, C_2C_3, C_3C_1. The author discusses the conformal invariants and obtains an inequality relation. The absolute invariant of a horn angle as previously defined is $I(1,2)=(\delta_2-\delta_1)/(\gamma_2-\gamma_1)^2$ where γ means curvature and $\delta=d\gamma/ds$. The fundamental theorem is $I(12)I(13)+I(23)I(21)+I(31)I(32)\geqq 0$. The *measure* of the horn angle is defined as $M(1,2)=$reciprocal of $I(1,2)=(\gamma_2-\gamma_1)^2/(\delta_2-\delta_1)$. The result is then $M(1,2)+M(2,3)+M(3,1)\geqq 0$ provided $M(1,2)M(2,3)M(3,1)<0$, that is, provided $(\delta_2-\delta_1)(\delta_3-\delta_2)(\delta_1-\delta_3)<0$. The equality sign holds only when the three curves are related so that the differences of the three δ's are proportional to the differences of the three curvatures γ. In particular this is valid when C_3 is obtained from C_1, C_2 by successive Schwarzian reflexion, or by conformal bisection as defined in the Proceedings of the International Congress of Mathematicians, Cambridge, 1912, p. 81. (Received January 30, 1936.)

102. Mr. R. B. Kershner: *On the addition of convex curves.*

Bohr (1913) has shown that the vectorial sum of n convex curves C_i is either a closed convex region or the ring-shaped domain between two convex curves. It is known that the supporting function of the outer curve is the sum of the supporting functions $h_i(\theta)$ of the C_i. In the present paper it is shown that if the inner curve exists then there must be one curve, say C_1, such that the supporting function of the inner curve is $h_1(\theta)-\sum_{i=2}^{n}h_i(\theta+\pi)$ save at most for the θ-intervals corresponding to the corners (if any) of the inner curve. This explicit result makes possible an analytical discussion of the inner curve similar to that given by Haviland (1933) for the outer curve. (Received January 22, 1936.)

103. Dr. D. H. Lehmer: *On the converse of Fermat's theorem.*

It has been known for more than a century that the statement (a) "n is a prime if it divides 2^n-2" suffers exceptions. In fact an infinite number of exceptions exist. Additional conditions on n so that (a) will be true have been given, but the best of these require the knowledge of a prime factor (the larger the better) of $n-1$. Another method of making (a) available as a test for primality, at least for a certain range of numbers, is to list all exceptions to (a) lying in this range together with their factorization. However, many of these exceptions are multiples of small primes so that their composite nature is readily discovered, and these may be disregarded. The writer has prepared a complete

list of these exceptions to (a) between 10^7 and 10^8 whose least prime factor exceeds 313, together with this factor. This list contains some 500 numbers and will appear in the American Mathematical Monthly. Twenty minutes suffice for determining by this method the prime or composite character of any 8-digit number. (Received January 25, 1936.)

104. Dr. D. H. Lehmer: *Polynomials for the n-ary composition of numerical functions.*

This paper deals with polynomials $\psi(x_1, x_2, \cdots, x_n)$ in n positive integers whose values are positive integers and which represent all positive integers, and are such that $\psi(x_1, x_2, \cdots, x_{n-1}, \psi(x_n, \cdots, x_{2n-1}))$ is a symmetric function of all of its $2n-1$ variables. Such polynomials would form a basis of the simplest sort of composition of numerical functions n at a time. All such polynomials are found and they constitute only a one parameter family. Moreover, each polynomial is the result of $n-2$ iterations of an appropriately chosen polynomial in 2 variables. This shows that n-ary composition with respect to polynomials may be achieved by repeated applications of the ordinary binary composition. (Received January 28, 1936.)

105. Mr. W. T. Puckett and Mr. G. C. Watson: *Concerning images of certain continua under monotone transformations.*

Let the continuum A having no continuum of condensation be decomposed into $A = F + \Sigma A_i$, where F is a closed and totally disconnected set and each A_i is a free arc having its endpoints in F. If $T(A) = B$ is a monotone transformation, then B takes the form $B = T(F) + \Sigma B_i$, where each B_i is a free arc having its endpoints in $T(F)$. Whence any continuum of condensation of B must be contained in $T(F)$. In case A is a linear graph the number of arcs B_i of B does not exceed the number of arcs A_i of A. In general, if, for any $b \in B$, $T^{-1}(b)$ contains no simple closed curve and at most one point of F, then B is homeomorphic with A. (Received January 29, 1936.)

106. Mr. E. D. Rainville: *On heat conduction in wedge shaped dams.*

Ordinarily temperature studies of concrete dams are based on approximations which replace the actual shape of the dam by a semi-infinite solid or by a slab of finite width. Often a wedge shaped figure is nearer to the actual case. In this paper certain problems of heat conduction in an infinite plane wedge are solved in two ways, one of which may be based upon Carslaw's point source function for such a wedge. A mean of practical importance is obtained and the formulas are simplified for computation purposes. The boundary and initial conditions satisfied are those best suited to the study of temperatures in many concrete dams. Only a finite portion of the wedge represents the dam; the remainder is treated as bedrock. An application to Boulder Dam is included. (Received January 10, 1936.)

107. Professor Francis Regan: *A note on a preceding paper concerning the admissibility of time series.*

In a paper (Transactions of this Society, vol. 36 (1934), no. 3) a lemma and a theorem due to Copeland are used in the development of the problem. In this note these two theorems are extended to apply in the field of geometrical probability. (Received January 15, 1936.)

108. Mr. L. B. Robinson: *A contribution to the theory of pseudo-functions.*

Recently the author proved that the solution of the Izumi equation $u'(x)=a(x)u(x^k)$, $(k=2, 3, 4, \cdots)$, was a pseudo-function, provided that $a(x)$ is an integral function with positive coefficients. He has succeeded now in demonstrating his theorem after lifting the restriction on $a(x)$. (Received January 24, 1936.)

109. Dr. Abraham Sinkov: *Necessary and sufficient conditions for generating the simple groups of order* 504 *and* 1092.

Necessary and sufficient conditions for generating the simple groups of orders 60, 168, and 660 by two operators of periods two and three are already known. No such definition is possible for the simple group of order 360. The purpose of the present study is to determine necessary and sufficient conditions for the two remaining simple groups whose orders do not exceed 1092. (Received January 21, 1936.)

110. Professor Morgan Ward and Mr. F. B. Fuller: *Continuous iteration of real functions.*

By an extension of a method developed by one of the authors (Ward, this Bulletin, vol. 40 (1934), no. 10) they obtain all the real continuous iterations of a given real continuous steadily increasing function $E(x)$ of the real variable x defined in the range $-\infty<a\leqq x<\infty$. By a continuous iteration of $E(x)$ is meant a real function $E_y(x)$ of the two real variables x and y such that $E_0(x)=x$, $E_1(x)=E(x)$; $E_{y+z}(x)=E_y(E_z(x))$; $E_y(a)$ steadily increasing and continuous in y in the range $0\leqq y\leqq 1$. The iteration is completely determined by the choice of $E_y(a)$ for $0\leqq y\leqq 1$, and the authors give the simple formulas which together enable them to exhibit $E_y(x)$ explicitly for $x\geqq a$, $y\geqq 0$. (Received January 11, 1936.)

111. Mr. R. Wilson: *A note on the asymptotic properties of orthogonal polynomials.*

Consider the system of orthogonal Tchebycheff polynomials $\phi_n(x)=x^n+\cdots$ corresponding to the finite interval (0, 1), and to a characteristic "S-function" $p(x)$. Making use of the recurrence-relation for $\phi_n(x)$, and of Poincaré's theorem, as supplemented by Perron, the author proves that $\lim\sup_{n\to\infty}[\phi_n(x)]^{1/n}$ for $0\leqq x\leqq 1$ is $1/4$, which is transfinite diameter of (0, 1). This supplements results announced earlier by J. Shohat for another type of $p(x)$ (American Journal of Mathematics, vol. 55 (1933), pp. 218–230). It follows that any function $f(x)$, analytic on (0, 1), can be expanded in a series according to the polynomials $\{\phi_n(x)\}$, which converges absolutely and uniformly for $0\leqq x\leqq 1$. (Received January 9, 1936.)

112. Professors Einar Hille and Otto Szász: *On the completeness of Lambert functions.*

Let $k_\beta(w)=\sum_1^\infty n^\beta w^n$, and let $S(\alpha, \beta, \lambda)$ denote the set of functions $(1-t)^\alpha k_\beta(t^{\lambda_n})$ where the λ's are complex numbers such that $\Re(\lambda_n)\geqq\delta>0$, and $\sum_1^\infty \Re(1/\lambda_n)=+\infty$. This set is complete in $L_p(0, 1)$, $1\leqq p<\infty$, if $\alpha>\max(\beta+1-1/p, -1/p)$. The set $S(\alpha, \beta, \lambda)+1$ is complete in $C[0, 1]$ if $\alpha>\max(\beta+1, 0)$. The analysis is based upon a use of the inversion formula of Möbius, and is capable of extensive generalizations. (Received February 17, 1936.)

113. Dr. Walter Leighton: *A continued fraction expansion.*

To every function $f(z)$ analytic in a region $R\supset z=0$ there corresponds a unique continued fraction expansion of the form (1): $c_0+\left[c_n z^{a_n}/1\right]_{n=1}^{n=\infty}$, where the a_n are positive integers and the c_n are constants. If $f(z)$ is a rational function of z, the continued fraction is terminating. If $f(z)$ is not rational, $c_n\neq 0$ $(n=1, 2, \cdots)$. The development (1) is the natural generalization of the classical expansion (2): $c_0+\left[c_n z/1\right]_{n=1}^{n=\infty}$ and avoids the introduction of the cumbersome conditions, inherent in (2), that $f(z)$ be "normal." Many of the classical theorems concerning (2) have valid analogues in the theory of (1). Several new convergence criteria are obtained. Example: If $\limsup |c_{2n+1}|=b_1$, $\liminf |c_{2n}|=b_0$, and $25b_1<b_0$, the continued fraction (2) converges except for poles in the annular region $b_1^{-1}>|4z|>25b_0^{-1}$. (Received February 25, 1936.)

114. Mr. R. P. Boas: *Some theorems on Fourier transforms and conjugate trigonometric integrals.*

Writing $f^*(x)=(1/\pi)\int_0^\infty [f(x+t)-f(x-t)][(\sin t)/t^2]dt$, the author shows that a necessary and sufficient condition that $f(x)\epsilon L^2(-\infty, \infty)$ should have a Fourier transform vanishing almost everywhere on $(-1, 1)$ is that $f^{**}(x)=-f(x)$ almost everywhere. He also considers $f^*(x)$ for a trigonometric Stieltjes integral, $f(x)=\int_{-R}^{R} e^{ixt}d\alpha(t)$, $\alpha(t)$ of bounded variation. If $\alpha(t)$ is constant on $(0, 1)$ and on $(-1, 0)$, $f^*(x)=-\tilde{f}(x)$, where $\tilde{f}(x)$ is the conjugate trigonometric integral. This relation yields an inequality for the conjugate of a trigonometric integral of the special form considered: $|\tilde{f}(x)|<(2/\pi)(2+\log R)$ if $|f(x)|\leqq 1$; $2/\pi$ is the best possible constant. This generalizes a result of M. Fekete for trigonometric polynomials. There is also obtained for trigonometric integrals (with $\alpha(t)$ not specialized on $(-1, 1)$) a generalization of the extension by G. Szegö (Schriften der Königsberger gelehrten Gesellschaft, vol. 5 (1928), pp. 59–70) of S. Bernstein's inequality for the derivative of a trigonometric polynomial: $R|\sigma(x)|+[f'(x)^2+\tilde{f}'(x)^2]^{(1/2)}\leqq R$ if $|f(x)|\leqq 1$, where $\sigma(x)$ is a certain arithmetic mean of $f(x)$. Similar inequalities involving mean values, suggested by a result of A. Zygmund, are also obtained. (Received February 20, 1936.)

115. Dr. Nathan Kaplan: *Einstein hypersurfaces of a euclidean space.*

In this paper, the properties of an Einstein hypersurface of m dimensions (V_m) which is immersed in an ordinary euclidean space of $m+1$ dimensions (R_{m+1}) are studied. The question of the existence of these hypersurfaces is left open. The following results are obtained: (1) this hypersurface can have at most

two distinct curvatures (l) and (k); (2) these are related by $(x-1)l+(m-x-1)k=0$ where (x) and $(m-x)$ are the number of curvatures equal to (l) and (k) respectively; (3) the (x) principal directions having (l) as curvature build a subspace V_x; (4) a similar result is valid for the remaining $(m-x)$ curvatures; (5) l and k are constants; (6) the subspaces V_x and V_{m-x} are of constant curvature and geodesic in V_m; and (7) these principal directions can always be selected such that they form an m-tuply orthogonal net. (Received February 20, 1936.)

116. Professor Marston Morse and Dr. Walter Leighton: *Singular quadratic functionals.*

The authors study the integral of a quadratic form in y' and y with coefficients which are continuous functions of x on an open interval, with singularities of various types at the ends of the interval. The coefficient of y'^2 is assumed positive. The study is made in the medium of Lebesgue integral theory and leads to important special cases of singular differential equations of the regular type. A new definition of conjugate points is required and new necessary and sufficient conditions. The results have applications in quantum mechanics and include as special cases certain fundamental integral inequalities found in chapter VII of the recent book of Hardy, Littlewood, and Pólya. (Received February 26, 1936.)

117. Professor V. G. Grove: *Differential geometry of a surface at a planar point.*

A point of a surface is called a planar point of the third order if every curve on the surface through the point has an inflexion at the point. The tangents to the curve of intersection of the surface and its tangent plane at the point determine a cubic involution. The author finds a canonical expansion of the surface in which one edge of the tetrahedron of reference giving rise to the expansion is a non-specialized tangent, and a canonical expansion in which two edges of the tetrahedron are the Hessian lines of the cubic involution. By means of the osculants of Bompiani for a plane curve at a point of inflexion, he discusses the various neighborhoods of arbitrary plane sections of the surface through the non-specialized tangent, and of sections through the Hessian lines of the cubic involution. Finally the loci of certain points connected with Bompiani's osculants for all sections of the surface at the planar point are discussed. (Received February 17, 1936.)

118. Dr. P. W. Ketchum: *On the expansion of a function holomorphic in distinct regions.*

A function $f(z)$ analytic at the distinct points $a_1, a_2, \cdots, a_n$, but not necessarily analytic in any region including two of these points, has a unique expansion in the series $\sum_{m=1}^{\infty} c_m F_m(z)$ where the functions $F_m(z)$ are polynomials defined by the equations $F_{pn+k}(z)=\Pi_{i=1}^{n}(z-a_i)^{p+1}/(z-a_k)$, $(k=1, 2, \cdots, n)$. This series converges absolutely and uniformly to $f(z)$ in and on any lemniscate $\Pi_{i=1}^{n}|z-a_i|=\eta$ which consists of n distinct ovals in each of which $f(z)$ is analytic. Generalizations of this theorem are also given. Proofs are very simple, in-

volving only a transformation of Bürmann's series. (Received February 17, 1936).

119. Professor Tibor Radó: *On transformations in the plane. I and II.*

If $u(x, y)$, $v(x, y)$ are single-valued and continuous in a closed square S, then the equations $u=u(x, y)$, $v=v(x, y)$ define a continuous transformation of S. Schauder (*Über stetige Abbildungen*, Fundamenta Mathematicae, vol. 12) obtained important results concerning the continuous transformations which are absolutely continuous in the sense of Banach. For the special case of transformations which satisfy a Lipschitz condition, Schauder derived a formula for the transformation of double integrals which has been applied recently in many investigations. On the other hand, for general absolutely continuous transformations Schauder had to make the additional restriction that a certain function $i(P)$ is bounded. The purpose of this paper is to remove this restriction and hence to make absolutely continuous transformations more easily manageable. The paper consists of two parts. The first part (*On continuous transformations*) contains a discussion of general continuous transformations and a proof that the restriction mentioned above is superfluous. In the second part (*On absolutely continuous transformations*) the formula for the transformation of double integrals is established, for absolutely continuous transformations, in complete generality, and semi-continuity properties are derived for certain integrals containing generalized Jacobians. The author plans to apply these results to the study of the area of surfaces in subsequent papers. (Received February 17, 1936.)

120. Professor R. D. Carmichael: *Proof that every positive integer is a sum of four squares.*

By means of certain reciprocal relations (essentially algebraic identities), which are of interest in themselves, the author gives a new proof of the stated theorem. The earlier proofs to which this one is most closely related are those of Euler and Dickson. (Received February 19, 1936.)

121. Professor R. D. Carmichael: *On numbers of the form* $a^2+\alpha b^2$.

Two elementary theorems concerning numbers of the given binary quadratic form are proved by very simple means and several elementary classic results are shown to follow immediately from these theorems. (Received February 19, 1936.)

122. Professor R. D. Carmichael: *On non-homogeneous linear differential equations of infinite order with constant coefficients.*

Using methods suggested by the stimulating memoir of A. Hurwitz (Acta Mathematica, vol. 20 (1897), pp. 285–312) and applying them to the indicated differential equations, the author obtains a series of results implying the existence under suitable hypotheses of several classes of integral solutions. (Received February 19, 1936.)

123. Professor R. D. Carmichael, Dr. W. T. Martin (National Research Fellow), and Dr. M. T. Bird: *On a classification of integral functions by means of certain invariant point properties.*

In the investigation of linear differential equations of infinite order and of certain other types of linear functional equations, the authors have severally had occasion to employ certain point properties of integral functions which remain unaltered in passing from one regular point to another. These invariant point properties are of considerable interest in themselves, and the theorems to which they give rise extend classic results in the theory of integral functions. In this paper the authors lay the foundations of a general theory of these invariant point properties and present the principal results which they have severally drawn upon in their recent investigations. (Received February 19, 1936.)

124. Professor Morris Marden: *An analogue to the Fekete theorem for the critical points of Green's function.*

Following a course parallel to that of his previous note on an analogue to the Grace-Heawood theorem (see abstract 42-1-72), the author finds the following as his principal result. "Let R_1 and R_2 be any two infinite regions with finite, non-intersecting boundaries B_1 and B_2, such that $G_1(x, y)$ and $G_2(x, y)$, the Green's functions (pole at ∞) for R_1 and R_2 respectively, have the same critical points. Let C_1 (center α_1, radius r_1) be any circle enclosing a closed branch of B_1 and let C_2 (center α_2, radius r_2) be any circle not overlapping C_1 but containing a closed branch of B_2. Then, if no critical point of the function $G(x, y)=[G_1(x, y)+G_2(x, y)]/2$ lies in the closed exterior of the hyperbola with foci at α_1 and α_2, and transverse axis of length (r_1+r_2), at least one critical point of $G(x, y)$—not a critical point of $G_1(x, y)$ and $G_2(x, y)$—lies in each interior of this hyperbola." This note will be published in combination with the previous one. (Received February 24, 1936.)

125. Professor Tibor Radó: *A lemma on the topological index.*

Let $C_k (k=0, 1, 2, \cdots)$ be a sequence of closed continuous curves given by equations $x=x_k(t)$, $y=y_k(t)$, where $x_k(t)$, $y_k(t)$ are continuous in $0\leqq t\leqq 1$ and satisfy $x_k(0)=x_k(1)$, $y_k(0)=y_k(1)$. Let $n_k(x, y)$ be defined as follows: If the point (x, y) is not on C_k, then $n_k(x, y)$ is equal to the topological index of (x, y) with respect to C_k; otherwise $n_k(x, y)=0$. Suppose all the functions $y_k(t)$ are of bounded variation in $0\leqq t\leqq 1$ and denote by T_k the total variation of $y_k(t)$. Then we have the following lemma: If $x_k(t)\to x_0(t)$, $y_k(t)\to y_0(t)$ uniformly in $0\leqq t\leqq 1$, and if $T_k\to T_0$, then $\iint |n(x, y)-n_k(x, y)|\,dx\,dy\to 0$. The paper concludes with a discussion of some corollaries and special cases, interesting either on account of their geometrical form or on account of applications to situations involving Jacobians. These applications will be developed in subsequent papers. (Received February 28, 1936.)

126. Professor Tibor Radó: *A remark on the area of surfaces.*

Let the continuous surface S be given by equations $x=x(u, v)$, $y=y(u, v)$, $z=z(u, v)$, $0\leqq u\leqq 1$, $0\leqq v\leqq 1$. Denote by $L(S)$ the area of S in the sense of

Lebesgue. For various special classes of surfaces it has been shown that $L(S)$ is given by the familiar integral formula. The most general result in this direction was obtained by McShane and Morrey. The purpose of this paper is to call attention to the following generalization of their result. In the definition of surfaces of class L (see Morrey, *A class of representations of manifolds*, American Journal of Mathematics, vol. 55 (1933)) replace condition (i) of Morrey ($x(u, v)$, $y(u, v)$, $z(u, v)$ absolutely continuous in the sense of Tonelli) by the assumption that $x(u, v)$, $y(u, v)$, $z(u, v)$ are of bounded variation in the sense of Tonelli. Call class L^* the class of surfaces determined by the definition thus modified. Then the area $L(S)$ of every surface S of class L^* is given by the familiar double integral (provided, of course, that we use for S a parametric representation with the properties specified in the definition of surfaces of class L^*). The proof depends upon certain simple properties of the topological index. (Received February 28, 1936.)

127. Dr. M. S. Webster: *On the relation of Appell polynomials to orthogonal polynomials.*

This note consists of a new, short proof of the recently established theorem that the system of Hermite polynomials is, except for a linear transformation on x, the only system of Appell polynomials which is also orthogonal. The proof is based upon the fundamental recurrence relation for orthogonal polynomials and the recurrence relation for Appell polynomials obtained by Sheffer. (Received February 28, 1936.)

128. Professor H. P. Thielman: *A generalization of the elementary transcendental functions by means of a theorem of Volterra.*

In this paper use is made of a theorem of Volterra for the derivation of transcendental functions from rational functions which vanish at infinity. The elementary transcendental functions are particular cases of the transcendentals so obtained. By applying the same process of transformation to these generalized transcendentals as was used for their derivation from rational functions, a larger class of transcendental functions is obtained, particular cases of which are Bessel's and Kelvin's functions of any order, and hypergeometric series. Many properties of these generalized transcendentals are then derived from the properties of the simpler functions which gave rise to them. The Laplace transformation is shown to be the inverse of one of the Volterra transformations here used. On the basis of this fact the asymptotic behavior of Bessel's and Kelvin's functions is studied and various infinite integrals of these functions are evaluated. (Received March 5, 1936.)

129. Dr. Rufus Oldenburger: *Ranks of 4-way matrices.*

With a given 4-way matrix $A=(a_{ijkl})$, $i, j, k, l=1, \cdots, n$, are associated 32 ranks which are defined in terms of the vanishing and non-vanishing of various determinant minors of A and matrices associated with A. These ranks divide into 6 essentially distinct sets according to the types of determinants in terms of which they are defined. In this paper there is determined the mini-

mum value of k for each set such that when k of the ranks in a given set are unity the remainder are also unity. It is also shown that when certain ranks of a set are unity ranks in other sets must also be unity. The paper was suggested by a property studied by Hitchcock and the author which states that if a 3-way rank of a 3-way matrix is unity, one of the 2-way ranks of this matrix is also unity. The ranks of the present paper are invariant under non-singular linear transformations in any field, and are useful in studying canonical quatrilinear forms. (Received March 6, 1936.)

130. Dr. Rufus Oldenburger: *Arithmetic invariants of binary cubic and binary trilinear forms.*

The canonical binary cubic forms x^3, x^3+y^3, x^2y of the binary cubic are well known. These cubics have been classified by means of algebraic invariants. In the present paper it is shown that these cubics can be distinguished more simply by means of a single arithmetic invariant called "factorization rank." For the above forms this rank has the values 1, 2, and 3 respectively. The rank is defined in terms of the orders of 2-way matrices which are factors of a 3-way matrix associated with the given cubic. This rank is invariant under non-singular linear transformations on the given cubic in any field. In this paper there are also evaluated the factorization ranks of the canonical binary trilinear forms in the complex field. (Received March 6, 1936.)

131. Dr. S. F. Barber: *Genus of an algebraic variety.*

Let V_r be an irreducible algebraic variety in a linear space S_{r+1} with a double variety D_{r-1} as its only singularities. Further, let V_r be a generic projection of a variety W_r in S_d, free from singularities. F. Severi in Rendiconti del Circolo Matematico di Palermo, vol. 28 (1909) has given a definition of arithmetic genus of V_r and a second definition of arithmetic genus of W_r and proved them to give the same value for $r=3$. This paper proves them equal for any r. (Received March 4, 1936.)

132. Mr. G. D. Nichols: *A generalized element of decomposition for doubly periodic functions.*

Starting from the integral first used by Teixeira, a derivation is given for a generalized element of decomposition for doubly periodic functions of the first, second, and third kinds. Relations between special cases of this element and the classical ones of Hermite and Appell are given. (Received March 5, 1936.)

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